

# An Alternate View on Strong Lexicalization in TAG

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TAG+12

A well-known fact

Lexicalized Grammars are *good* for parsing algorithms

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## Idea

- generalize TAGs to multi-dimensional TAGs;
- lexicalization via increase in dimensionality:  
⇒ every  $d$ -TAG is strongly lexicalized by some  $(d+1)$ -TAG

- 1 Introduction
  - Lexicalization
  - Existing Results
- 2 Preliminaries
  - Adjunction & Substitution
  - TAGs as 3-d trees
  - TAGs as multi-dimensional structures
- 3 Strong Lexicalization
  - $d$ -TAGs are  $(d+1)$ -TSGs
  - $d$ -TAGs strongly lexicalize  $d$ -TSGs
- 4 Conclusion

A grammar is lexicalized if the atoms from which compound structures are assembled each contain a pronounced lexical item.

Lexicalized grammars are *finitely ambiguous*:

- recognition is decidable;
- parsing is simplified [Schabes et al., 1988]

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Derive lexicalization properties of TAGs by generalizing to multidimensional structures

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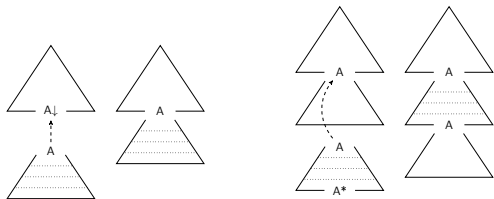
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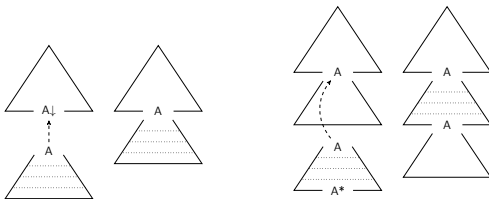


Substitution can be regarded as adjunction of a **footless** tree at a leaf node

## Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite leaf nodes

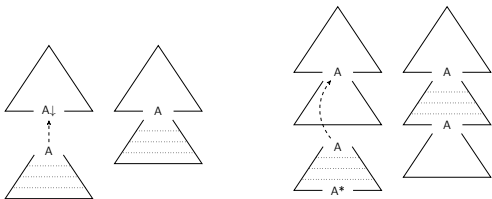
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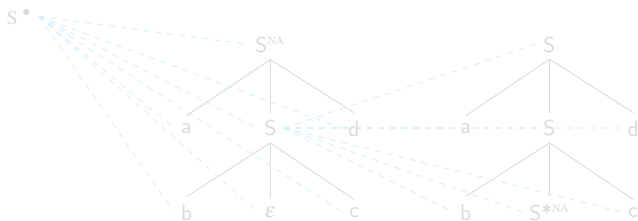
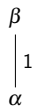
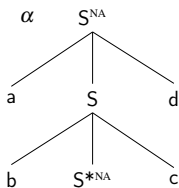
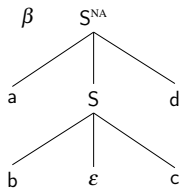


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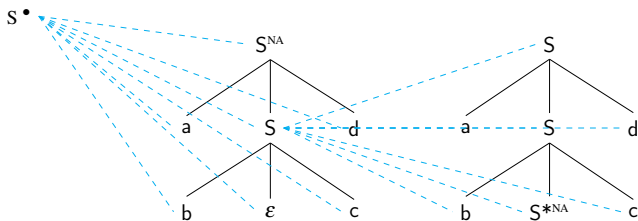
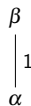
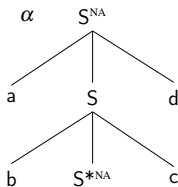
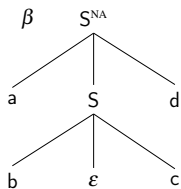
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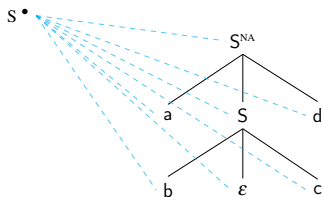


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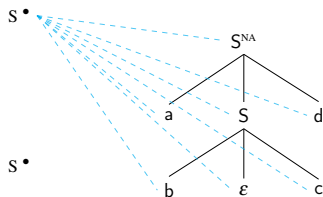
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d-dimensional Local Structure

- d-dimensional mother
- $y^d$ : (d)-yield

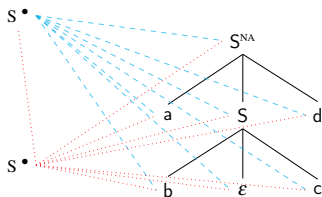
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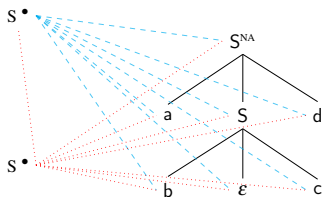
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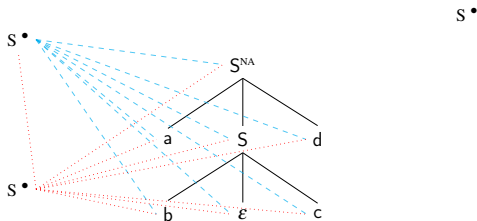
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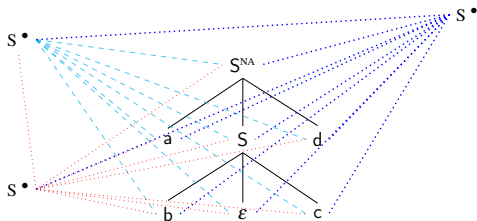
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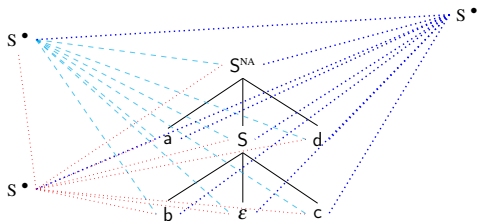
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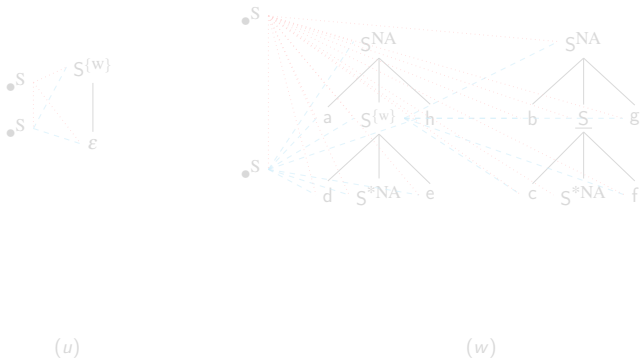


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# A 4d Example: the 8-language

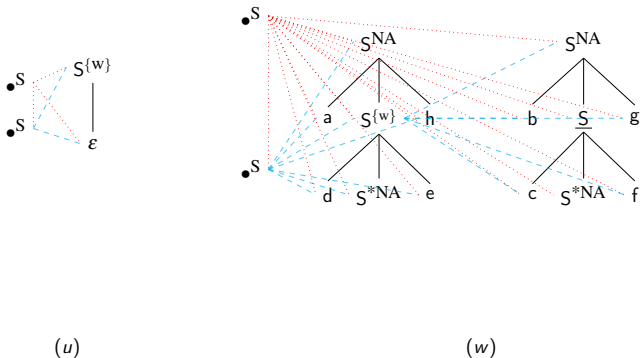
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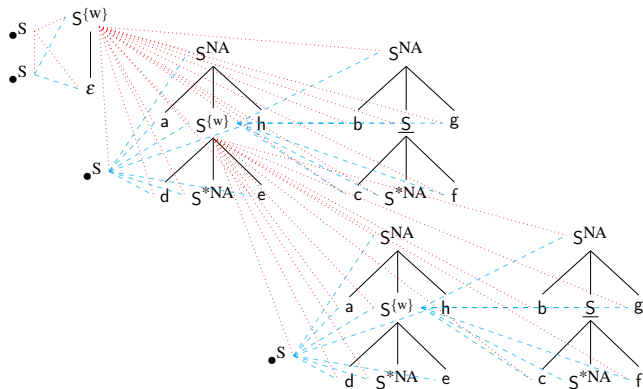
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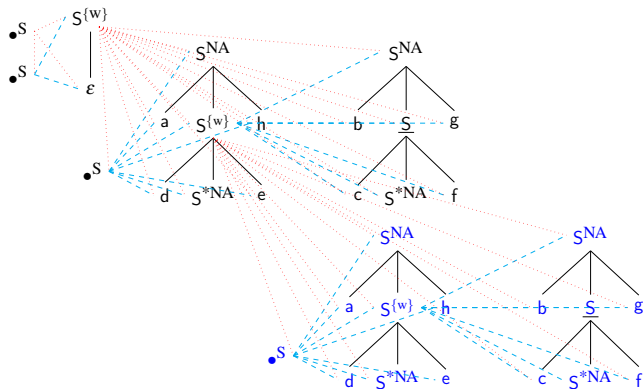
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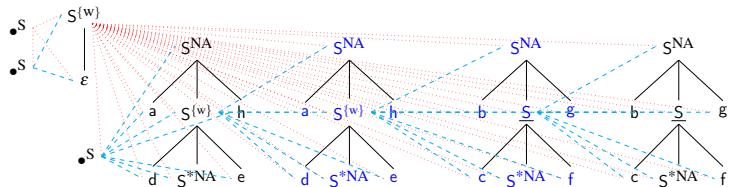
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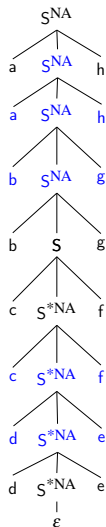






# The 8-language: a derivation

The 2-d yield!





## The Road so far

- Substitution as Adjunction
- TAGs as natural 3-d structures
- the generalization to higher dimensions is easy

## An Important Concept

d-dimensional Local Structure:

- d-dimensional mother;
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## Next Steps

- can we get new insights about TAG mechanisms?
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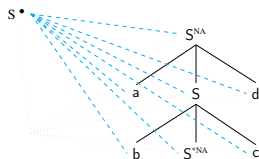
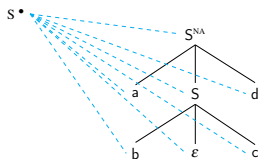
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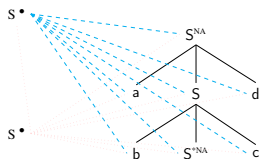
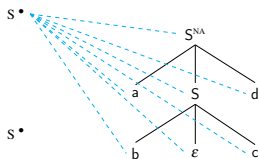
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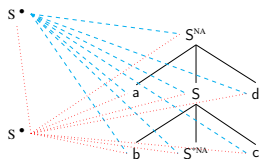
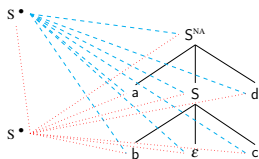
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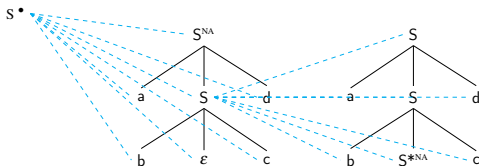
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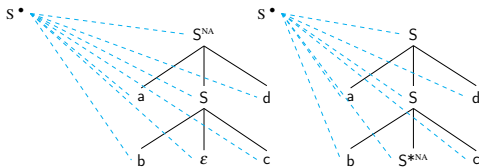


We can show that adjunction in  $d$  is substitution in  $(d+1)$



Adjunction in 3d / Substitution in 4d

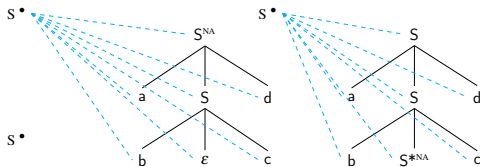
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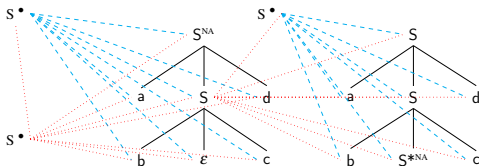


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[Schabes, 1990]

TAGs strongly lexicalize TSGs.

## A Lexicalization Procedure

Consider a TSG  $G$ :

- 1 Divide  $G$  in recursive and non-recursive;
- 2 Construct the set  $I_{lex}$  of initial trees;
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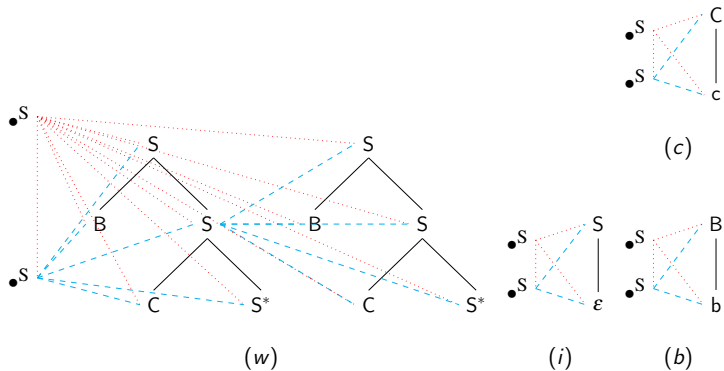
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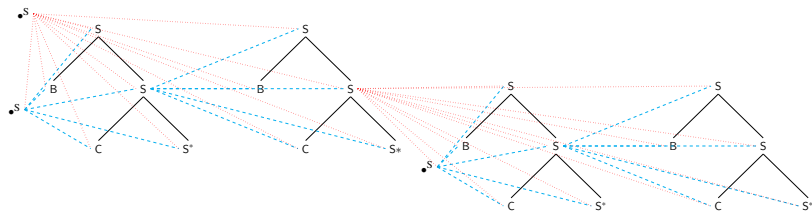
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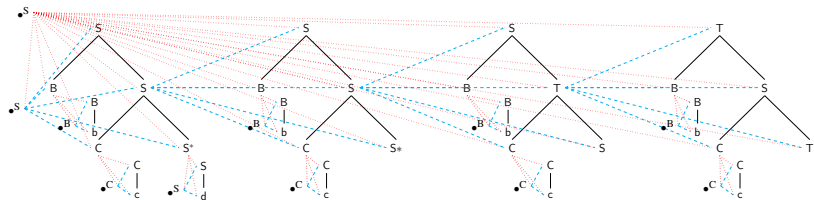






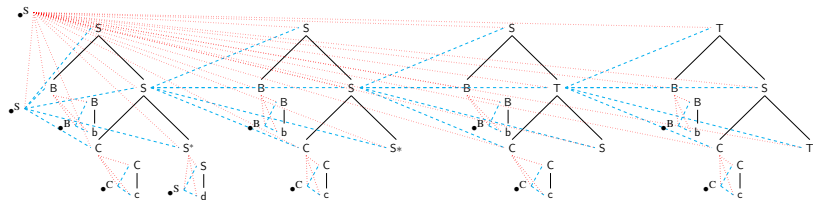
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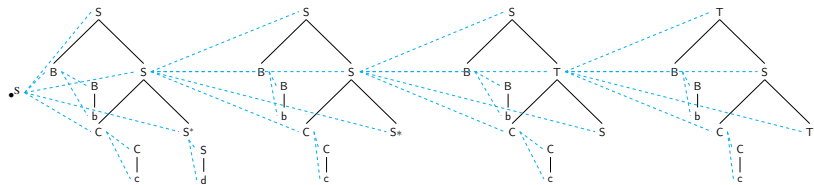


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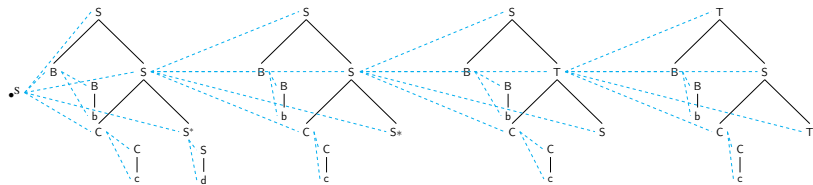
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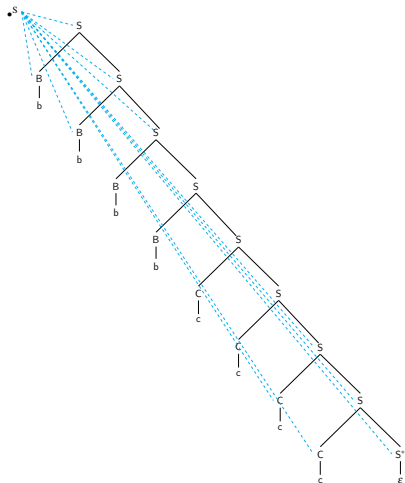


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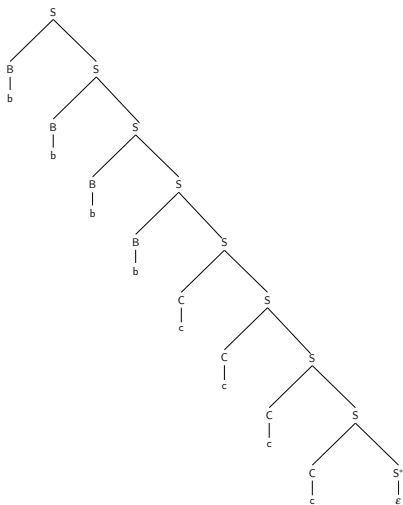
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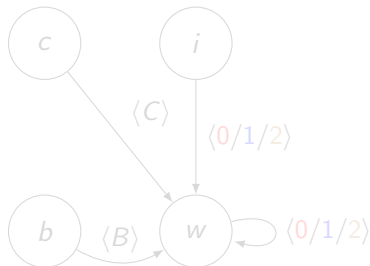
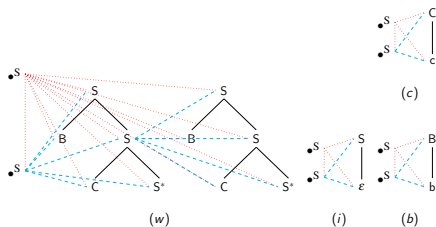
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# Lexicalization of a $d$ -TSG: Step 1

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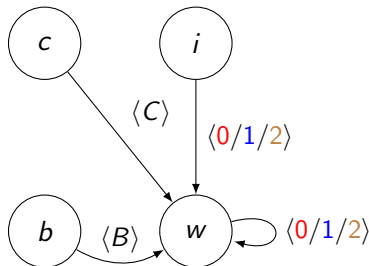
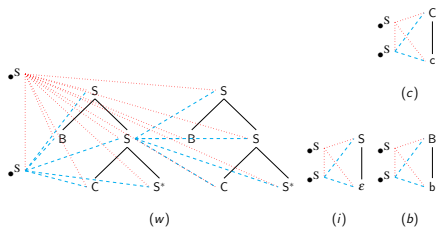
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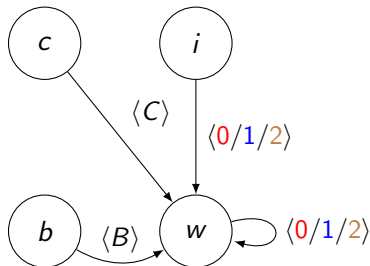
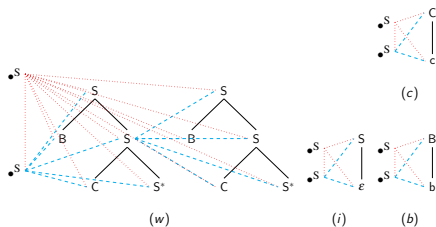
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Step 2: Determine the set  $I_{lex}$ .

- $T(NR)$  : the closure of  $NR$  under adjunction

$I_{lex}$

is the maximal subset of  $T(NR)$  that only contains  $d$ -trees whose root is labeled by the start category  $S$ .

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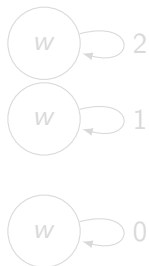
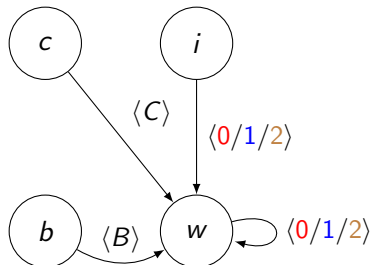
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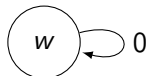
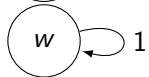
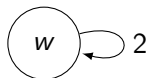
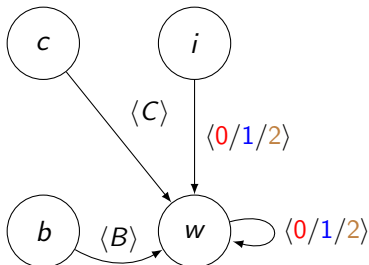
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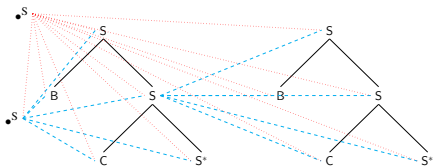
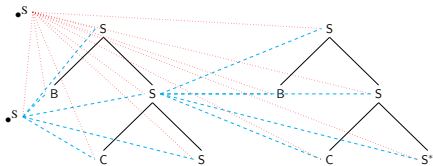
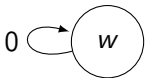


# Lexicalization of a $d$ -TSG: Step 4

## Step 4:

### Determine $A_{lex}$

- expand base cycles;
- relabel  $3d$  foot node;
- exhaustive substitution;

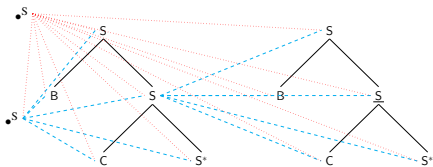
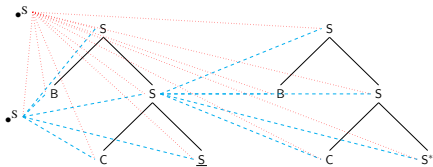


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## Step 4:

### Determine $A_{lex}$

- expand base cycles;
- relabel  $3d$  foot node;
- exhaustive substitution;



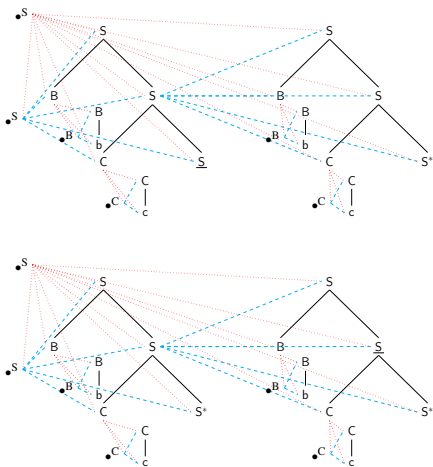
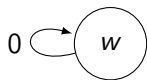


# Lexicalization of a $d$ -TSG: Step 4

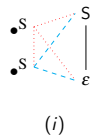
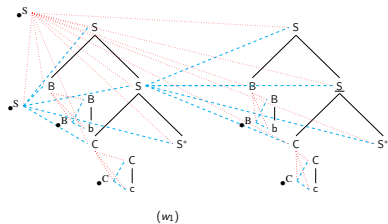
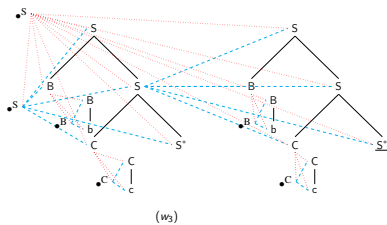
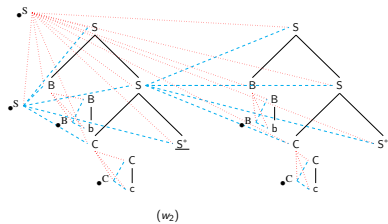
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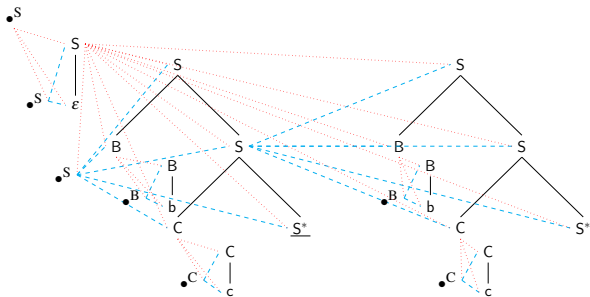
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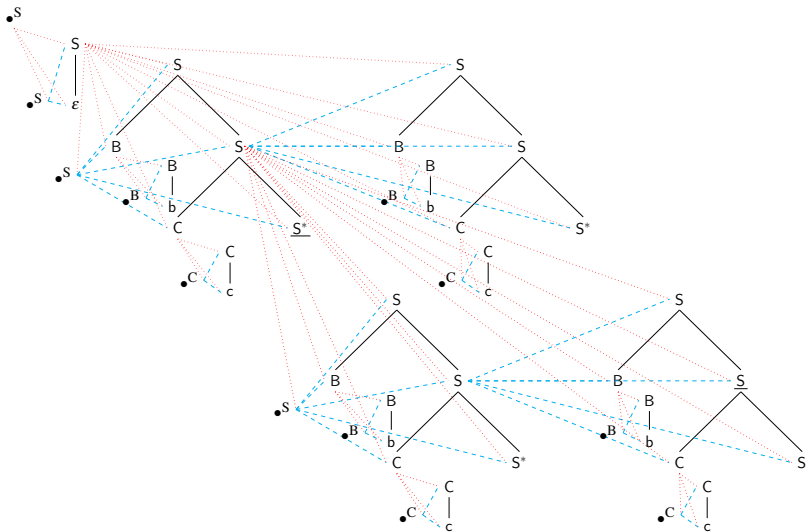
# The lexicalized d-TAG



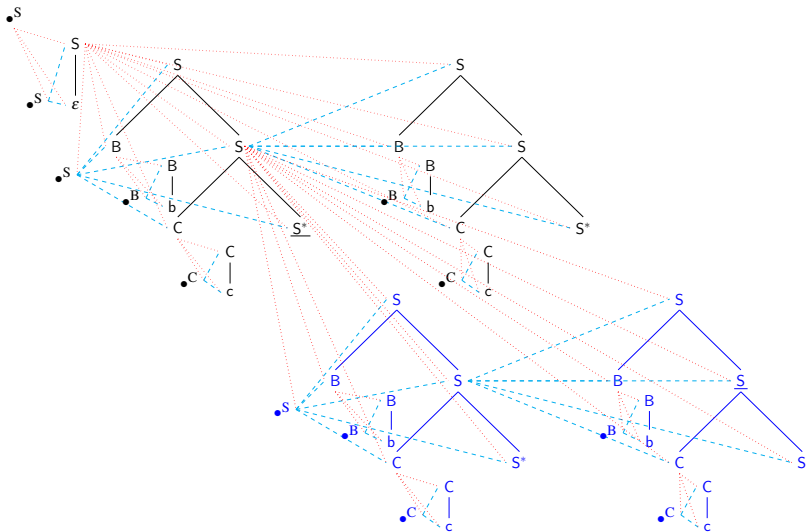
# The lexicalized d-TAG: a derivation



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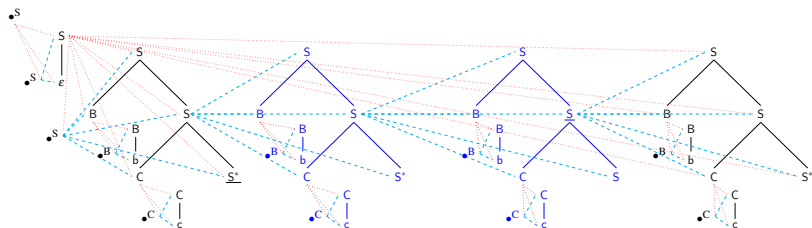


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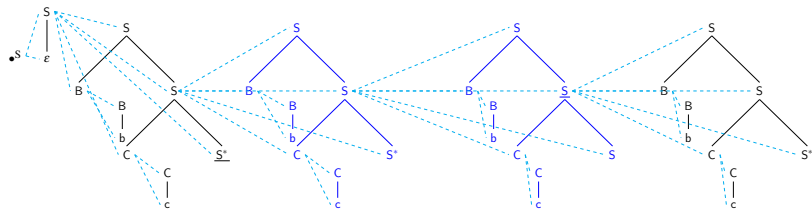


# The lexicalized d-TAG: a derivation

The 3d yield ...

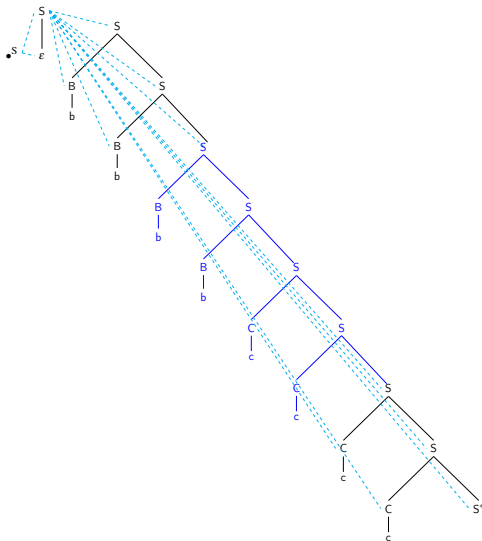


## The 3d yield!



# The lexicalized d-TAG: a derivation

The 2d yield ...







## Proposition

For each finitely ambiguous  **$d$ -dimensional** TSG that does not generate the empty string and contains only useful trees, there is a strongly equivalent  **$d$ -dimensional** Lexicalized TAG.

but

$d$ -TSGs are equivalent to  $(d - 1)$ -TAGs.

## Proposition

$(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

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- TAGs can be generalized to higher dimensional trees [Rogers, 2003]
- TAGs strongly lexicalize CFGs/TSGs [Schabes, 1990]

⇒  $(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## TAGs as higher dimensional-trees

- lifting of existing results is straightforward
- increase in generative power
- what about parsing?

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Fujiyoshi, A. (2004).

Epsilon-free grammars and lexicalized grammars that generate the class of the mildly contextsensitive languages.

*In Proceedings of the 7th International Workshop on Tree Adjoining Grammar and Related Formalisms*, pages 16–23.



Kuhlmann, M. and Satta, G. (2012).

Tree-adjoining grammars are not closed under strong lexicalization.

*Computational Linguistics*, 38:617–629.



Maletti, A. and Engelfriet, J. (2012).

Strong lexicalization of tree adjoining grammars.

*In Proceedings of the 50th Annual Meeting of the Association for Computational Linguistics: Long Papers - Volume 1, ACL '12*, pages 506–515.



Rogers, J. (1998).

On defining TALs with logical constraints.

In Abeillé, A., Becker, T., Rambow, O., Satta, G., and Vijay-Shanker, K., editors, *Fourth International Workshop on Tree Adjoining Grammars and Related Frameworks (TAG+4)*, pages 151–154.



Rogers, J. (2003).

Syntactic structures as multi-dimensional trees.

*Research on Language and Computation*, 1:265–305.



Schabes, Y. (1990).

*Mathematical and Computational Aspects of Lexicalized Grammars*.

PhD thesis, Philadelphia, PA, USA.





Schabes, Y., Abeillé, A., and Joshi, A. K. (1988).

Parsing strategies with 'lexicalized' grammars: Application to tree adjoining grammars.

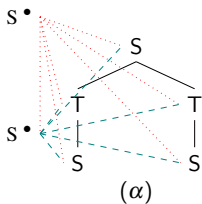
Technical Report MS-CIS-88-65, Department of Computer & Information Science, University of Pennsylvania, Philadelphia, PA.

# One final question

Are  $d$ -dimensional TAGs closed under strong lexicalization?

[Kuhlmann and Satta, 2012]

TAGs are not closed under strong lexicalization



$$\begin{aligned} (\alpha) \quad S^{NA} &- \left( \begin{array}{c} \bullet S \\ S^{NA} \\ S^{OA} \\ T^{NA} \\ S^{OA} \\ T^{NA} \end{array} \right) - S^{NA} \\ (\beta) \quad S^{NA} &- \left( \begin{array}{c} \bullet S \\ S^{NA} \\ a \end{array} \right) - S^{NA} \\ (\gamma) \quad S^{OA} &- \varepsilon \end{aligned}$$

*Excess*

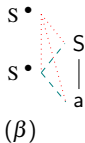
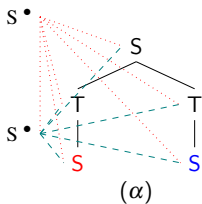
measures the distance between a root node and a terminal node

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$$(\alpha) \quad S^{NA} - \left( \begin{array}{c} \bullet S \\ S^{NA} \\ S^{OA} \\ T^{NA} \\ S^{OA} \\ T^{NA} \end{array} \right) - S^{NA}$$

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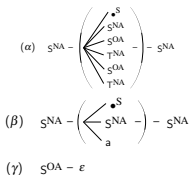
$$(\gamma) \quad S^{OA} - \varepsilon$$

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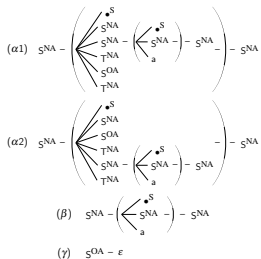
# $d$ -TAGs are not closed under strong lexicalization

## Non Lexicalized



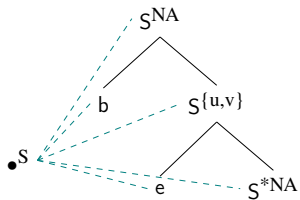
*max. excess* of node  $a$  is  
**unbounded**

## Lexicalized

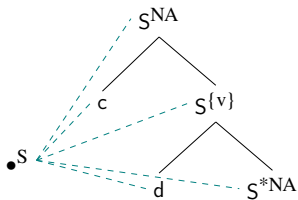


*max excess* of node  $a$  is  
**2**

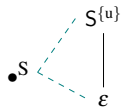
# Increasing the power by increasing the dimensionality: Ex. 2



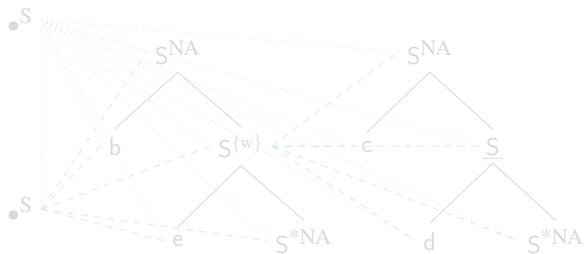
$u$



$v$



$i$

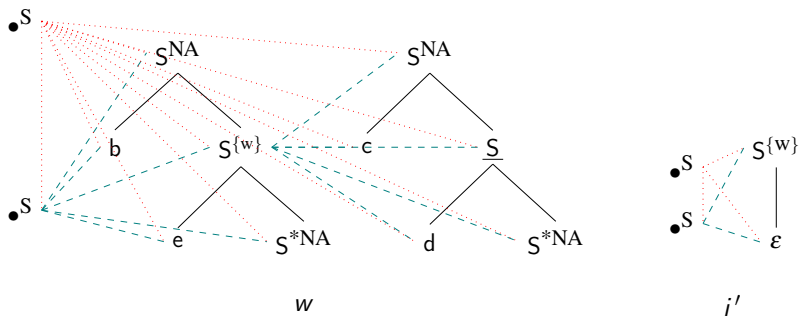
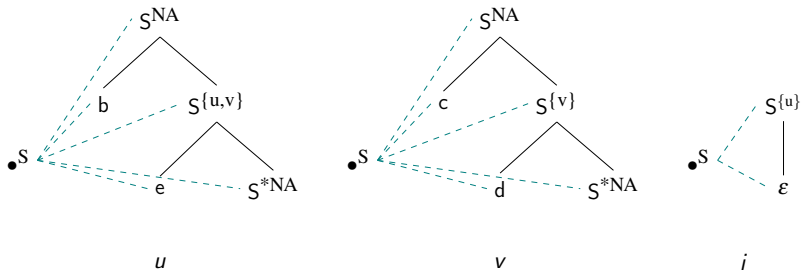


$w$



$i'$

# Increasing the power by increasing the dimensionality: Ex. 2



# Map of Existing Results

