



# Mathematical Linguistics & Typological Complexity

**Aniello De Santo**

`aniellodesanto.github.io`  
`aniello.desanto@utah.edu`  
`@AnyDs`

SIGTYP Lecture Series  
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# (Some) Big Questions

- ▶ Are there **laws** that govern linguistic knowledge?
- ▶ **Why** are those the laws?
- ▶ Do they relate to **typological gaps**, i.e. logically possible patterns we don't (seem to) find?
- ▶ What can we infer about **human learning processes**?

## Cross-disciplinarity for the win

- ▶ Stand on the shoulders of giants.
- ▶ Cross-fertilization and multiple explanatory levels.
- ▶ Yields new generalizations and data.

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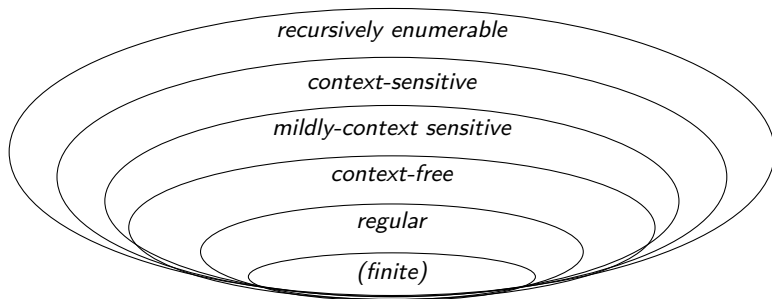
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- 1 Linguistics and Formal Language Theory
- 2 Refining the Hierarchy via Typological Insights
- 3 Artificial Grammar Learning
- 4 Summing Up & Future Directions

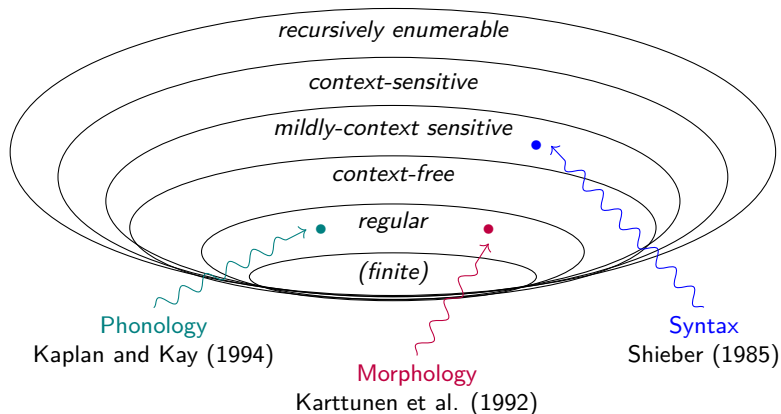
# Computational Theories of Language

Languages (stringsets) can be classified according to the complexity of the grammars that generate them.

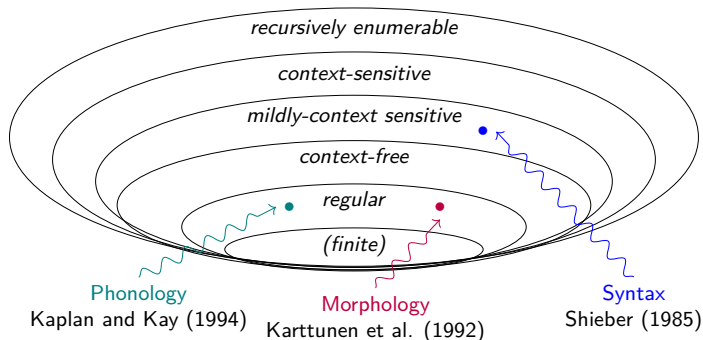


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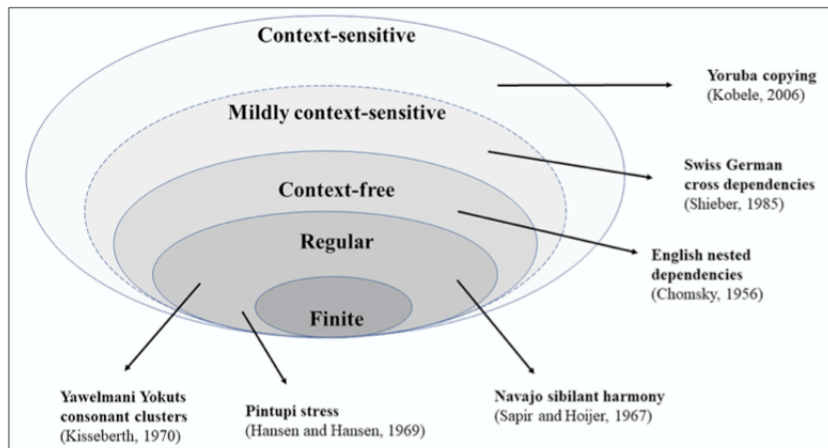
# Precise Theories $\Rightarrow$ Precise Predictions



## Precise predictions for:

- ▶ typology  $\rightarrow$  e.g. no center embedding in phonology
- ▶ learnability  $\rightarrow$  e.g. no Gold learning for regular languages
- ▶ cognition  $\rightarrow$  e.g. finitely bounded working memory

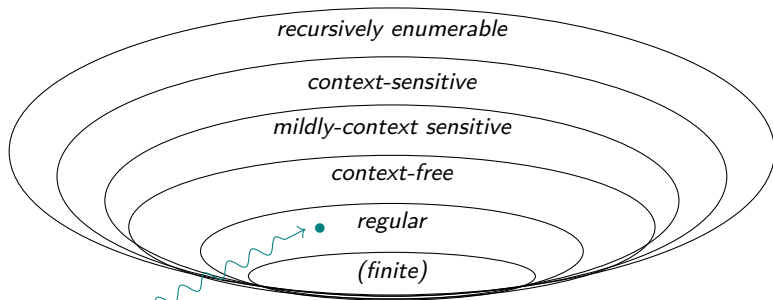
# Classifying Patterns



**Figure 1:** The Chomsky Hierarchy. Various features of natural language occupy different regions of the hierarchy. Figure reproduced from Figure 1 in Heinz (2010: 634) with permission.



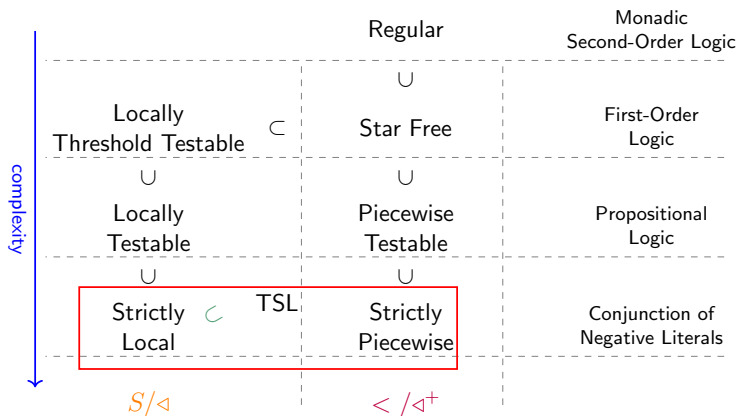
# Phonology as a Regular System



Phonology

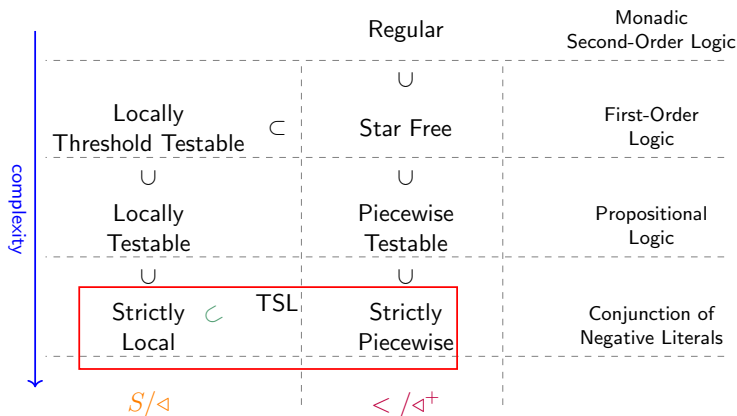
Kaplan and Kay (1994)

# Beyond Monolithic Classes: Subregular Languages



- Multiple equivalent characterizations:  
 $\Rightarrow$  algebraic, logic, automata...

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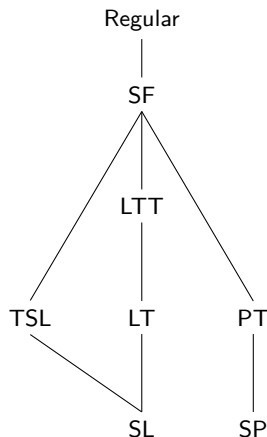
# Phonology as a Subregular System

## Subregular Phonotactics

- ▶ Majority of phonological patterns are **subregular** (Heinz 2011a,b; Chandlee 2014; Graf 2017:a.o.).

*Most phonological and morphological rules correspond to p-subsequential relations.*

*(Mohri 1997)*



# Phonology as a Subregular System

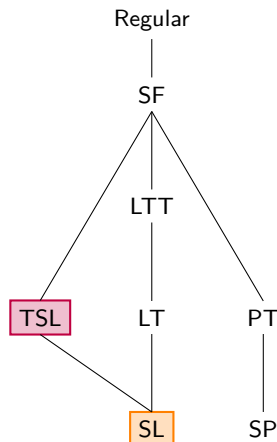
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*Most phonological and morphological rules correspond to p-subsequential relations.*

*(Mohri 1997)*

A caveat:  
Mostly phonotactics today!



# Local Dependencies in Phonology

## 1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. \*rad
- b. rat

## 1 Intervocalic voicing

Forbid voiceless segments in between two vowels

- (2) a. \*faser
- b. fazer

These patterns can be described by **strictly local** (SL) constraints.

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# Local Dependencies in Phonology are SL

## Example: Word-final devoicing

- ▶ Forbid voiced segments at the end of a word:  $*[+voice]\$$
- ▶ **German:**  $*z\$, *v\$, *d\%$  ( $\$$  = word edge).

\$ r a **d** \$                      \$ r a t \$

## Example: Intervocalic voicing

- ▶ Forbid voiceless segments in-between two vowels:  $*V[-voice]V$
- ▶ **German:**  $*ase, *ise, *ese, *isi, \dots$

\$ f a **s** e r \$                      \$ f a z e r \$



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\*     $\$$    r   a   d    $\$$      *ok*     $\$$    r   a   t    $\$$

## Example: Intervocalic voicing

- ▶ Forbid voiceless segments in-between two vowels:  $*V[-voice]V$
- ▶ **German:**  $*a\textcolor{red}{s}e, *i\textcolor{red}{s}e, *e\textcolor{red}{s}e, *i\textcolor{red}{s}i, \dots$

$\$$    f   a   s   e   r    $\$$

$\$$    f   a   z   e   r    $\$$

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\*      $\$$    f   a   s   e   r    $\$$      *ok*      $\$$    f   a   z   e   r    $\$$

# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. \* ha<sup>s</sup>xintilawa<sup>f</sup>  
b. \* ha<sup>f</sup>xintilawa<sup>s</sup>  
c. ha<sup>f</sup>xintilawa<sup>f</sup>

Example: Samala

\*\$ ha<sup>s</sup>xintilawa<sup>f</sup>\$

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► **But:** Sibilants can be arbitrarily far away from each other!

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► **But:** Sibilants can be arbitrarily far away from each other!

\* \$<sup>s</sup>tajanowonwa<sup>f</sup>\$



# Locality Over Tiers

\*\$**s**tajano**n**wa**j**\$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size  $n$ ;

## Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶ Projection of selected segments on a tier  $T$  (Goldsmith 1976)
- ▶ Strictly local constraints over  $T$  determine wellformedness
- ▶ Unbounded dependencies are local over **tiers**



# Locality Over Tiers

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# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (4) a. \* ha<sup>s</sup>xintilaw<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>

- ▶ What do we need to project? [+strident]
- ▶ What do we need to ban? \*[+ant][−ant], \*[−ant][+ant]

I.E. \*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʒ, \*<sup>ʒ</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>                      ʃ

.....

\* \$ha<sup>s</sup>xintilaw<sup>ʃ</sup>\$

ʃ                      ʃ

.....

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>\$

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I.E. \*<sup>s</sup><sub>f</sub>, \*<sup>s</sup><sub>3</sub>, \*<sup>z</sup><sub>f</sub>, \*<sup>z</sup><sub>3</sub>, \*<sup>f</sup><sub>s</sub>, \*<sup>3</sup><sub>s</sub>, \*<sup>f</sup><sub>z</sub>, \*<sup>3</sup><sub>z</sub>

## Example: TSL Samala

\* \$ha<sup>s</sup>xintilaw<sup>f</sup>\$

<sup>ok</sup> \$ha<sup>f</sup>xintilaw<sup>f</sup>\$

# Interim Summary: SL and TSL for Phonology

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Provably correct and efficient learning algorithms (Jardine and McMullin 2017)
- ▶ Rules out unattested patterns (cf. Lai 2015, Aksenova et al. 2016, Graf & De Santo 2019, a.o.)

## But:

- ▶ Typological variation is complex, knowledge is limited
- ▶ Can we truly gain cognitive insights?

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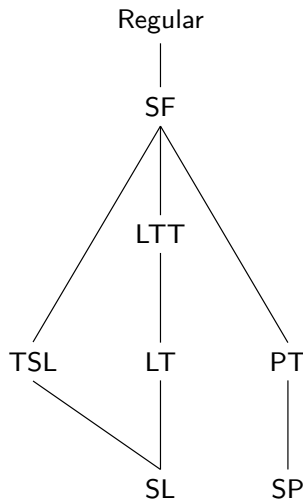
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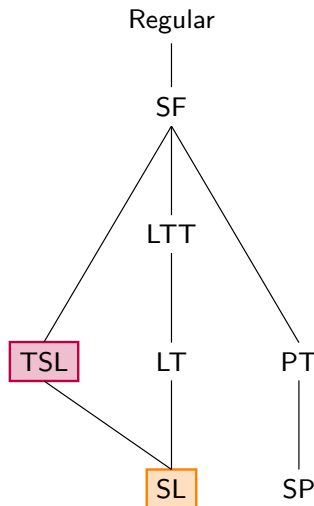
# SL and TSL: So What?



- **But** not every long-distance pattern is TSL!  
(McMullin 2016, Mayer & Major 2018, De Santo & Graf 2019)



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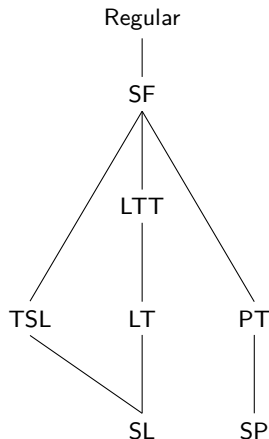


- **But** not every long-distance pattern is TSL!  
(McMullin 2016, Mayer & Major 2018, De Santo & Graf 2019)

# Concurrent Processes (De Santo and Graf, 2019)

## Observation

- TSL is not closed under intersection



- We want to also account for multiple processes  
So we can cover the complete phonotactics of a language
- Multiple non-interacting processes in attested patterns

# A TSL Outlier

## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin2016)

1) Underlying causative prefix /s(:)-/

	<i>Base</i>	<i>Causative</i>	
--	-------------	------------------	--

a.	uga	s:-uga	"be evacuated"
----	-----	--------	----------------

b.	a <sup>s</sup> :twa	s-a <sup>s</sup> :twa	"settle, be levelled"
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- |    |                 |                           |                       |
|----|-----------------|---------------------------|-----------------------|
| a. | uga             | <b>s</b> :-uga            | "be evacuated"        |
| b. | a <b>s</b> :twa | <b>s</b> -a <b>s</b> :twa | "settle, be levelled" |

### 2) Sibilant harmony

*Base*      *Causative*

- |    |              |                         |                                |
|----|--------------|-------------------------|--------------------------------|
| a. | fiaʃr        | ʃ- fiaʃr                | "be full of straw, of discord" |
| b. | n <b>z</b> a | <b>z</b> :-n <b>z</b> a | "be sold"                      |

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### 3) Sibilant voicing harmony blocked

*Base*      *Causative*

- |    |                 |                        |                         |
|----|-----------------|------------------------|-------------------------|
| a. | uk <b>z</b>     | <b>s</b> :-uk <b>z</b> | "recognize"             |
| b. | q:u <b>z</b> :i | ʃ- q:u <b>z</b> :i     | "be dislocated, broken" |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ} \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*s\text{ʃ}, {}^*\text{ʒ}s, {}^*\text{ʃ}s, {}^*zs, {}^*z\text{ʃ}, {}^*z\text{ʒ}, {}^*\text{ʃ}z, {}^*\text{ʃ}\text{ʒ}, {}^*\text{ʒ}\text{ʃ}, {}^*\text{ʒ}z \}$$

\* z m: ʒ d a w l

*ok* ʒ m: ʒ d a w l

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z      ʒ

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\*  
z      ʒ  
 .....  
 \*

z m: ʒ d a w l

ʒ      ʒ  
 .....  
 ok ʒ m: ʒ d a w l

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\* z      ʒ

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ok ʒ      ʒ

ok ʒ m: ʒ d a w l

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ, s, z, ʃ, q} \}$$

$$S = \{ *sʒ, *sz, *sʃ, *ʒs, *ʃs, *zs, *zʃ, *zʒ, *ʃz, *ʃʒ, *ʒʃ, *ʒz \}$$

*ok* ʃ q u ʒ: i

\* s q u ʒ: i

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$$T = \{ \text{ʒ}, \text{s}, \text{z}, \text{ʃ}, \text{q} \}$$

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ʃ q ʒ:  
 .....  
 ok [ʃ] [q] u [ʒ:] i

\* s q u ʒ: i

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ok

ʃ q ʒ

ok ʃ q u ʒ i

\* s q u ʒ i

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<sup>ok</sup>  
<sup>ok</sup> ʃ q ʒ:  
 .....  
<sup>ok</sup> ʃ q u ʒ: i

s q ʒ:  
 .....  
 \* [s] [q] u [ʒ:] i

# Sibilant Harmony in IMDLAWN TASHLHIYT

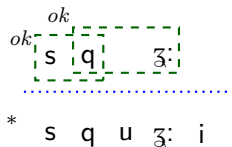
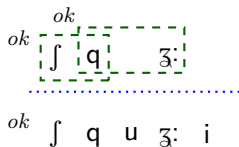
## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, \text{s}, \text{z}, \text{ʃ}, \text{q} \}$$

$$S = \{ {}^*\text{sʒ}, {}^*\text{sz}, {}^*\text{sʃ}, {}^*\text{ʒs}, {}^*\text{ʃs}, {}^*\text{zs}, {}^*\text{zʃ}, {}^*\text{zʒ}, {}^*\text{ʃz}, {}^*\text{ʃʒ}, {}^*\text{ʒʃ}, {}^*\text{ʒz} \}$$



# Sibilant Harmony in IMDLAWN TASHLHIYT

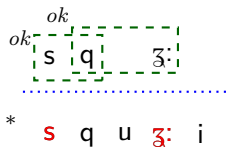
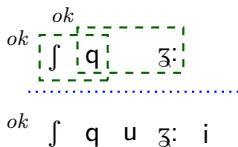
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Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, \text{s}, \text{z}, \text{ʃ}, \text{q} \}$$

$$S = \{ * \text{sʒ}, * \text{sz}, * \text{sʃ}, * \text{ʒs}, * \text{ʃs}, * \text{zs}, * \text{zʃ}, * \text{zʒ}, * \text{ʃz}, * \text{ʃʒ}, * \text{ʒʃ}, * \text{ʒz} \}$$





# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{ʃ}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{ʃ}z, ^*\text{ʃ}\text{ʒ}, ^*\text{ʒ}\text{ʃ}\}$$

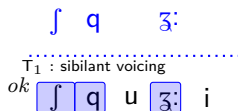
*ok*    ʃ    q    u    ʒ:    i

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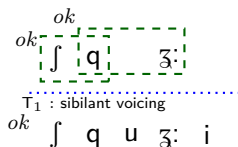
$ok$  ʃ q ʒ:  
 $T_1$  : sibilant voicing  
 $ok$  ʃ q u ʒ: i

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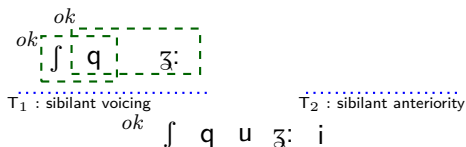
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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, \text{ʃ}\} \quad S_2 = \{^*s\text{ʒ}, ^*s\text{ʃ}, ^*\text{ʒ}s, ^*\text{ʃ}s, ^*zs, ^*z\text{ʃ}, ^*z\text{ʒ}, ^*\text{ʃ}z, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

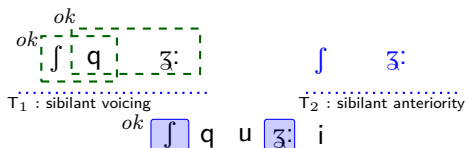
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \int, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\int z, ^*\int\text{ʒ}, ^*\text{ʒ}\int\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, \int\} \quad S_2 = \{^*s\text{ʒ}, ^*s\int, ^*\text{ʒ}s, ^*\int s, ^*zs, ^*z\int, ^*z\text{ʒ}, ^*\int z, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

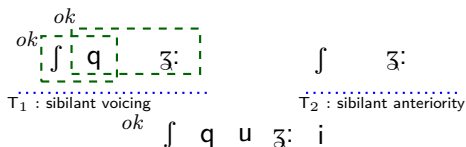
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \int, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\int z, ^*\int\text{ʒ}, ^*\text{ʒ}\int\}$$

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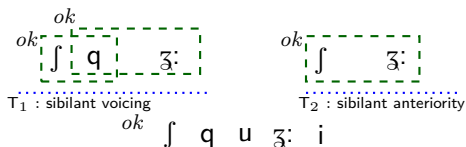
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# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{ʃ}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{ʃ}z, ^*\text{ʃ}\text{ʒ}, ^*\text{ʒ}\text{ʃ}\}$$

Unbounded agreement in anteriority:

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\* s q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

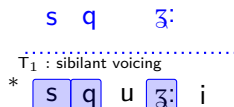
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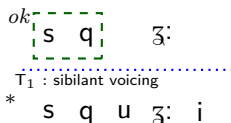
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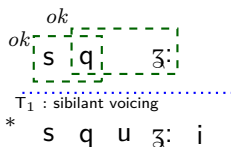
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# Multi-Tier Strictly Local (MTSL) Languages (2/2)

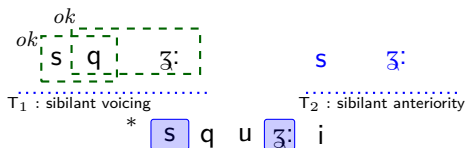
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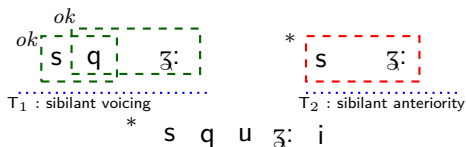
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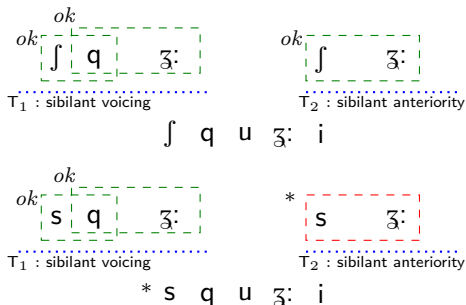
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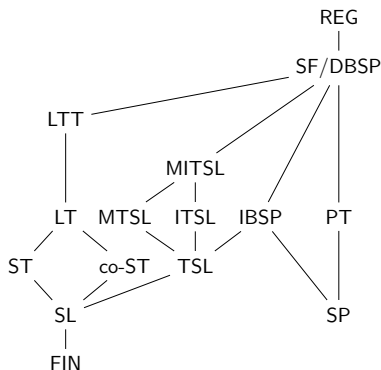
# Accounting for Concurrent Processes

- MTSL: TSL closure under intersection  
(De Santo & Graf, 2019)



- Intersection closure accounts for multiple concurrent processes
- Can characterize the complete phonotactics of a language

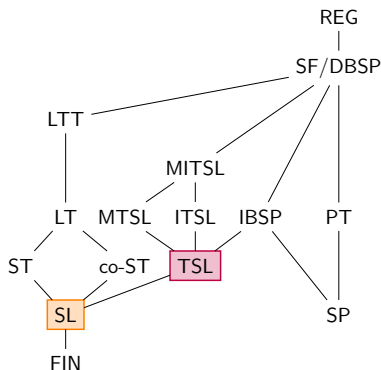
# A Plethora of Combination



- ▶ The goal is **not** identifying a single “correct” class
- ▶ Pinpoint fundamental properties of the patterns:  
SL:  $\triangleleft$  , TSL:  $\triangleleft_T$  , ...

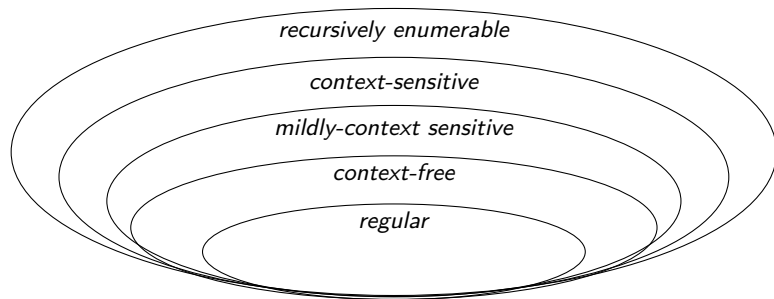


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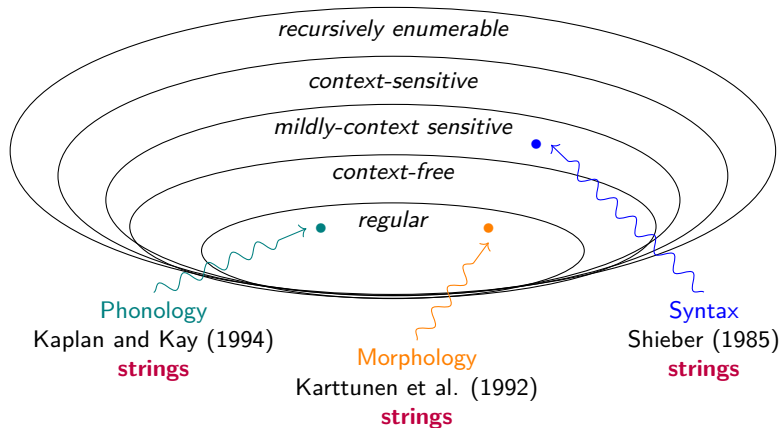


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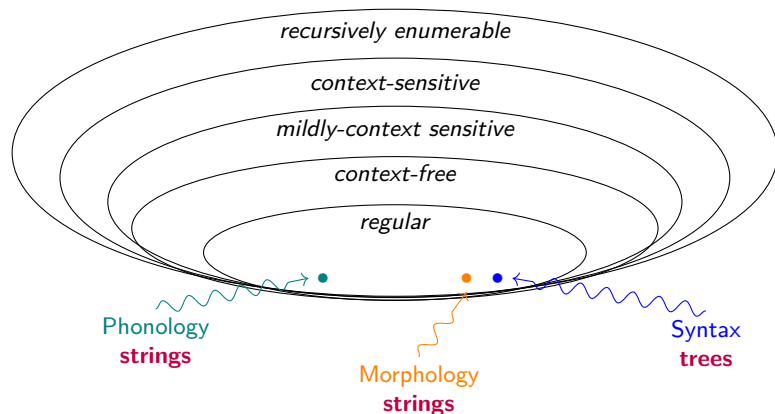
# Cross-domain Parallels



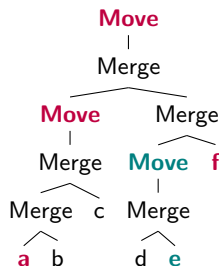
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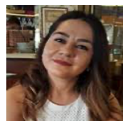
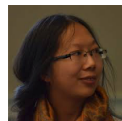
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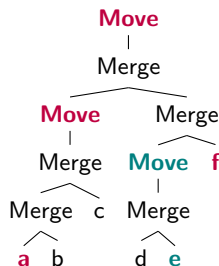
# Subregular Syntax



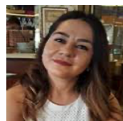
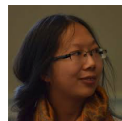
- ▶ Some results for syntax
  - ▶ regular tree languages (Michaelis 2004; Kobele et al. 2007)
  - ▶ subregular **operations** (Graf 2018)
  - ▶ subregular **dependencies/constraints** (Laszakovits 2018; Vu et al. 2019)
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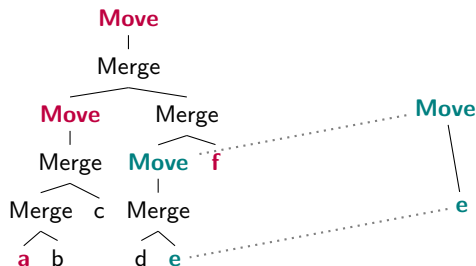
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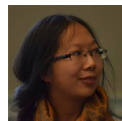
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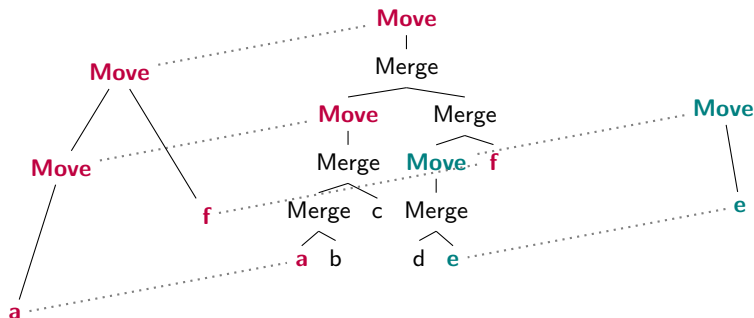
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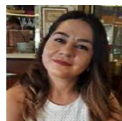
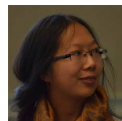
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# Interim Summary: Again, So What?

## Strong Parallelism

Subregular dependencies in phonology, (morphology), and syntax  
**subregular** over their respective **structural representations**.

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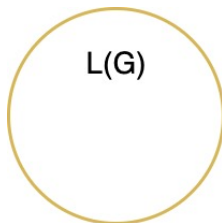
- ▶ Typological variation is complex
- ▶ Our knowledge of attested pattern is limited

# Outline

- 1 Linguistics and Formal Language Theory
- 2 Refining the Hierarchy via Typological Insights
- 3 Artificial Grammar Learning**
- 4 Summing Up & Future Directions

# Artificial Grammar Learning (AGL)

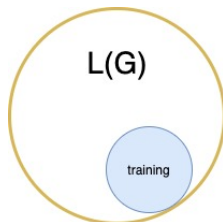
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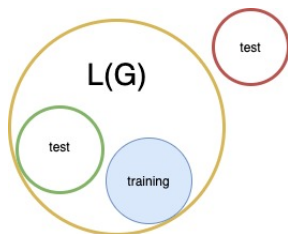
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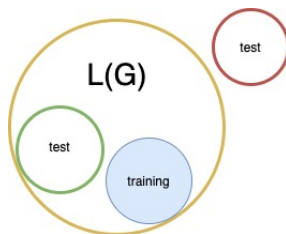


- ▶ Possible vs. impossible rules (Musso et al. 01, Culbertson 21)
- ▶ Child language acquisition (Nowal and Baggio 2017, a.o.)  
→ but careful with test sets (De Santo 2017)
- ▶ Animal cognition (Wilson et al. 2020, a.o.)  
→ cf. (De Santo and Rawski 2020)



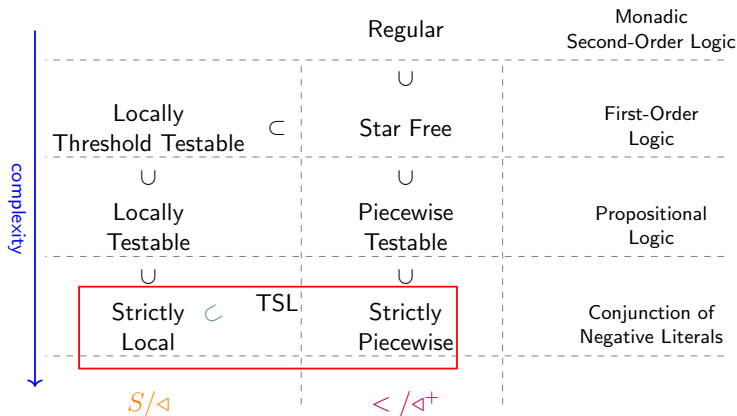
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# Testing Subregular Predictions




# Example: Attested vs. Unattested Patterns

## Attested: Unbounded Sibilant Harmony


- ▶ Every sibilant needs to harmonize

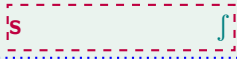
  
 \* \$ha **s**xintilawʃ\$

  
<sup>ok</sup> \$haʃxintilawʃ\$

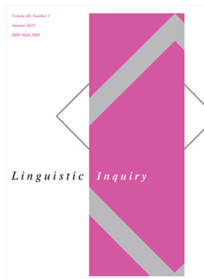
## Unattested: First-Last Harmony

- ▶ Harmony only holds between initial and final segments

  
<sup>ok</sup> \$ha **s**xintilawʃ\$

  
 \* \$ **s**atxintilawʃ\$

# Lai (2015)



## Learnable vs. Unlearnable Harmony Patterns

Regine Lai

Posted Online July 09, 2015

[https://doi.org/10.1162/LING\\_a\\_00188](https://doi.org/10.1162/LING_a_00188)

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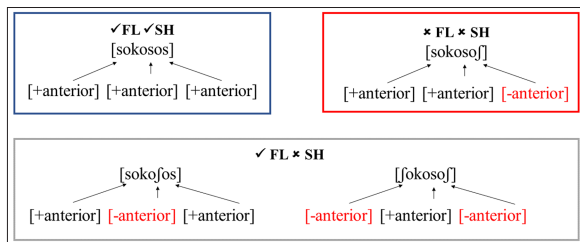
**Linguistic Inquiry**

Volume 46 | Issue 3 | Summer 2015

p.425-451

**Keywords:** phonotactics, learnability, computational phonology, formal theory, typology, dependencies

## Lai (2015): Stimuli



**Figure 3:** Comparison of SH and FL stimuli.

## Lai (2015): Stimuli

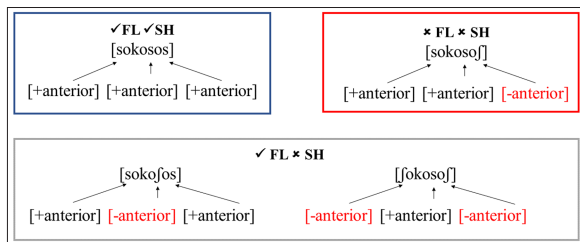


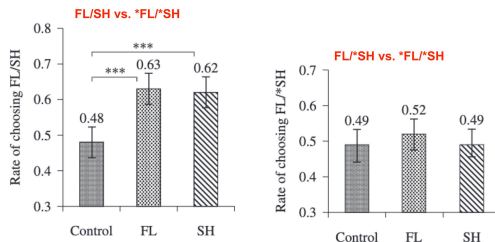
Figure 3: Comparison of SH and FL stimuli.

Table 6

Predicted results with respect to the control group for each test pairing if Sibillant Harmony and First-Last Assimilation grammars were internalized

Conditions	Pairs		
	FL/*SH vs. *FL/*SH (e.g., [s ... ʃ ... s] vs. [s ... s ... ʃ]) Rate of FL/*SH	FL/SH vs. *FL/*SH (e.g., [s ... s ... s] vs. [s ... s ... ʃ]) Rate of FL/SH	FL/SH vs. FL/*SH (e.g., [s ... s ... s] vs. [s ... ʃ ... s]) Rate of FL/SH
SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

# Lai (2015): Results



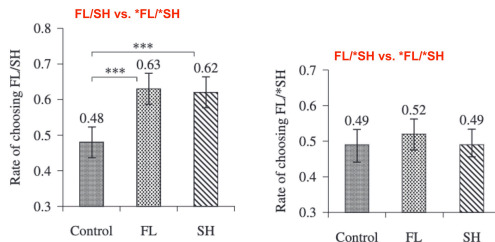
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► See Avcu and Hestvik (2020), Avcu et al. (2019) for replications

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SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

► See Avcu and Hestvik (2020), Avcu et al. (2019) for replications



# A Plethora of Testable Predictions

## Observation

- ▶ Attested patterns **A** and **B** are TSL.
- ▶ But combined pattern **A+B** is not TSL.

## Prediction

- ▶ **A+B** should be harder to learn than **A** and **B**

# Example: Compounding Markers

## Morphotactics as Tier-Based Strictly Local Dependencies

Alëna Aksënova   Thomas Graf   Sedigheh Moradi

- ▶ Russian has an infix **-o-** that may occur between parts of compounds.
- ▶ Turkish has a single suffix **-sı** that occurs at end of compounds.

(5) vod   **-o-**   voz   **-o-**   voz  
water -COMP- carry -COMP- carry  
'carrier of water-carriers'

(6) türk   bahçe   kapı   **-sı**   (\***-sı**)  
turkish garden gate -COMP (\*-COMP)  
'Turkish garden gate'



## Example: Compounding Markers [cont.]

- ▶ Russian and Turkish are TSL.

	<b>Tier<sub>1</sub></b>	COMP affix and stem edges #
<b>Russian</b>	<i>n</i> -grams	oo, \$o, o\$
<b>Turkish</b>	<i>n</i> -grams	sisi, \$si, si#

- ▶ The combined pattern would yield **Ruskish**: stem<sup>*n*+1</sup>-si<sup>*n*</sup>
- ▶ This pattern is not regular and hence **not TSL either**.

### Testable Predictions

- ▶ Can naive subjects learn Russian-like, Turkis-like, and Ruskish-like compounding?

# Outline

- 1 Linguistics and Formal Language Theory
- 2 Refining the Hierarchy via Typological Insights
- 3 Artificial Grammar Learning
- 4 Summing Up & Future Directions**

# Of Black Swans and Flying Pigs



# Of Black Swans and Flying Pigs



# Of Black Swans and Flying Pigs



- ▶ Not a single data point, but classes of phenomena
- ▶ Value of restrictive theories: predictive and explanatory
- ▶ We learn from falsifying them too!

# Complexity as a Magnifying Lens

- ▶ We can compare patterns and predictions across classes
- ▶ We can also compare patterns within a same class

## Proceedings of the Society for Computation in Linguistics

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Volume 1

Article 8

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2018

## Formal Restrictions On Multiple Tiers

Alena Aksenova

*Stony Brook University*, [alena.aksenova@stonybrook.edu](mailto:alena.aksenova@stonybrook.edu)

Sanket Deshmukh

*Stony Brook University*, [sanket.deshmukh@stonybrook.edu](mailto:sanket.deshmukh@stonybrook.edu)

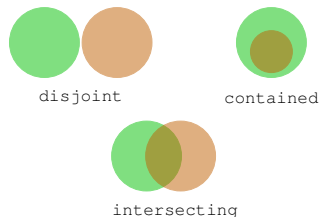
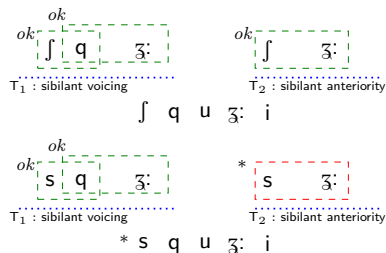




# Testing Harmony Systems

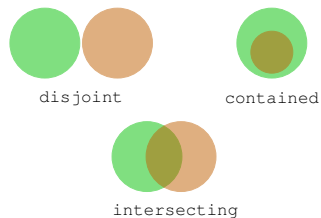
## Reminder:

- MTSL's multiple-tier idea...

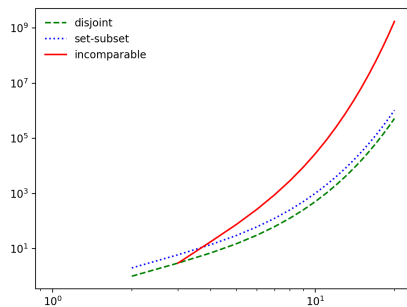
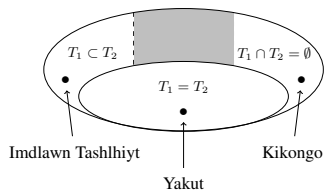


**Figure 2:** Theoretically possible tier alphabet relations

# Testing Harmony Systems (cont.)



**Figure 2:** Theoretically possible tier alphabet relations



**Figure 7:** Growth of number of partitions of sets containing up to 20 elements (log log scale)

# Learnability Generalizations

## Learning Interactions of Local and Non-Local Phonotactic Constraints from Positive Input

Aniello De Santo  
Dept. of Linguistics  
University of Utah

aniello.desanto@utah.edu

Alëna Aksënova  
Google NYC  
alenaks@google.com

- ▶ Efficiently learn  $\text{MITSL}_2^2$  grammars from positive data

## Unlearnable Patterns

- ▶ No overlapping tiers with the same  $^*\rho_1\rho_2$  restriction  
e.g.  $T_1 = \{a, b, c\}$ ,  $T_2 = \{a, b, d\}$ ,  $G_1 = G_2 = \{^*ab\}$
- ▶ This is *predicted* from the structure of the grammar  
(see also Lambert et al. 2021)

# From Blackbox to Blackbox

## Multi-Element Long Distance Dependencies: Using SPk Languages to Explore the Characteristics of Long-Distance Dependencies

**Abhijit Mahalunkar**

Applied Intelligence Research Center  
Technological University Dublin  
Dublin, Ireland

abhijit.mahalunkar@mydit.ie

**John D. Kelleher**

ADAPT Research Center  
Technological University Dublin  
Dublin, Ireland

john.d.kelleher@dit.ie

- ▶ Strictly-piecewise Languages
  - ▶ Basically: Skip-gram models
  - ▶ Capture long distance dependencies over strings
  - ▶ Modulate parameters of variation:  
e.g., length of the dependency,  
alphabet size, etc.

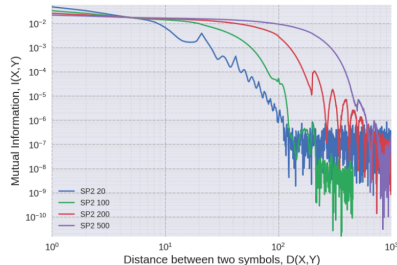


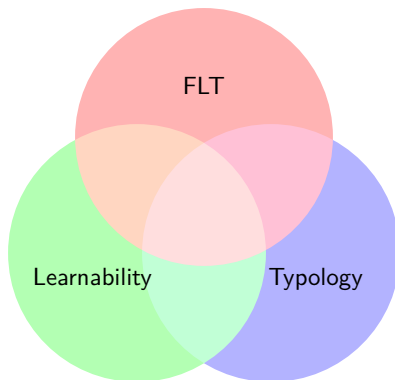
Figure 3: LDD characteristics of datasets of SP2 grammar exhibiting LDDs of length 20, 100, 200 and 500.

# Theory Building

*The problem that we cannot deduce [...] theories from data is a limitation, or **perhaps an attribute**, of all empirical science [...] Still, one may abduce hypotheses [...] Abduction is **reasoning from observations** [...] It consists of two steps: generating candidate **hypotheses** (abduction proper), and selecting the “best” explanatory one (inference to the **best explanation**).*

*(van Rooij & Baggio 2020, pg. 9)*

# A Collaborative Enterprise!



# Thank you!



**Mathematical Linguistics and Cognitive Complexity**

Aniello De Santo, Jonathan Rawski

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# From Blackbox to Blackbox

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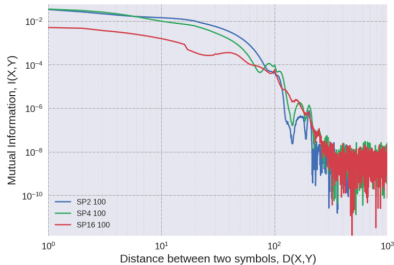


Figure 2: LDD characteristics of datasets of SP2, SP4 and SP16 grammar exhibiting LDD of length 100.

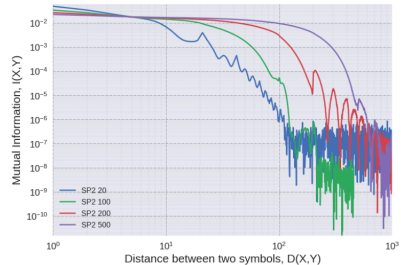


Figure 3: LDD characteristics of datasets of SP2 grammar exhibiting LDDs of length 20, 100, 200 and 500.

## Example: Circumfixation in Indonesian

- ▶ Indonesian has circumfixation with no upper bound on the distance between the two parts of the circumfix.

(7) maha siswa  
big pupil  
'student'

(8) \*(ke-) maha siswa \*(-an)  
NMN- big pupil -NMN  
'student affairs'

- ▶ Requirements: exactly one *ke-* and exactly one *-an*

<b>Tier<sub>1</sub></b>	contains all NMN affixes	\$	an			ke	ke	\$
<b>Tier<sub>0</sub></b>	contains all morphemes							
<i>n</i> -grams	\$an, ke\$, keke, anan	\$	an	m	s	ke	ke	\$

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<i>n</i> -grams	\$an, ke\$, keke, anan	\$	an	m	s	ke	ke	\$

## Example: Swahili *vyo*

Swahili *vyo* is **either a prefix or a suffix**,  
depending on presence of negation. (?)

- (9) a.      a-                  vi-                  **soma -vyo**  
                 SBJ:CL.1- OBJ:CL.8- read   -REL:CL.8  
                 'reads'
- b.      a-                  si-                  **vyo-**                  vi-                  **soma**  
                 SBJ:CL.1- NEG- REL:CL.8- read -OBJ:CL.8  
                 'doesn't read'

# Example: Swahili *vyo* [cont.]

- (10) a. \* a- **vyo-** vi- **soma**  
 SBJ:CL.1- REL:CL.8- OBJ:CL.8- read
- b. \* a- **vyo-** vi- **soma -vyo**  
 SBJ:CL.1- REL:CL.8- OBJ:CL.8- read -REL:CL.8
- c. \* a- si- **vyo-** vi- **soma**  
 SBJ:CL.1- NEG- REL:CL.8- OBJ:CL.8- read  
**-vyo**  
 REL:CL.8-
- d. \* a- si- vi- **soma -vyo**  
 SBJ:CL.1- NEG- OBJ:CL.8- read REL:CL.8-

# Example: Swahili *vyo* [cont.]

## Generalizations About *vyo*

- ▶ may occur at most once
- ▶ must follow negation prefix *si-* if present
- ▶ is a prefix iff *si-* is present

**Tier<sub>1</sub>** contains *vyo*, *si*, and stem edges #

**Tier<sub>0</sub>** contains all morphemes

*n*-grams **vyovyo**, **vyo##vyo** “at most one *vyo*”

**vyosi**, **vyo##si** “*vyo* follows *si*”

**si##vyo**, **\$vyo##** “*vyo* is prefix iff *si* present”

# TSL Phonology: Accounting for Context

## ► **Unbounded Tone Plateauing in Luganda (UTP)**

No L may occur within an interval spanned by H.

(Hyman 2011)

- (11) a.    **L****H**LLLL  
      b.    LLLL**H**L  
      c.    \* **L****H**LL**H**L  
      d.    **L**HHHH**L**

Example



## TSL Phonology: Accounting for Context

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### Example

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 c. \***L****H**LL**H****L**  
 d. **L****H**HH**H****L**

### Example

L **H** L L **H** L  
 .....  
 \* L **H** L L **H** L

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## ► **Unbounded Tone Plateauing in Luganda (UTP)**

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 b. **LLLLHL**  
 c. \* **LHLLHL**  
 d. **LHHHHL**

### Example

**LHLLHL**  
 .....  
 \* **LHLLHL**

# Input-Sensitive TSL (ITSL) Languages

## Defining Tier Projection

Tier projection controlled by:

**1** label of segment

TSL

**1**

TSL languages are characterized by:

- ▶ a **1**-local projection function;
- ▶ strictly  $k$ -local constraints applied on  $T$ .

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**Idea:**

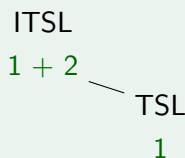
- ▶ Projection is an input-strictly local transduction (Chandley 2015)
- ▶ **What if:** the locality of  $E_T$  was higher than 1?

# Input-Sensitive TSL (ITSL) Languages

## Defining Tier Projection

Tier projection controlled by:

- 1 label of segment
- 2 local context



TSL languages are characterized by:

- ▶ a 1-local projection function;
- ▶ strictly  $k$ -local constraints applied on  $T$ .

### Idea:

- ▶ Projection is an input-strictly local transduction (Chandlee 2015)
- ▶ **What if:** the locality of  $E_T$  was higher than 1?

# Accounting for Context [cont.]

**A ITSL analysis for UTP** (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

## Example

*ok* **L H L L L L**

*\** **L H L L H L**



# Accounting for Context [cont.]

**A ITSL analysis for UTP** (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

## Example

**H**  
.....  
*ok* **L****H** **L** **L** **L** **L**

\* **L** **H** **L** **L** **H** **L**

# Accounting for Context [cont.]

## A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
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### Example

**H L**  
.....  
*ok* **L** **H L** **L L L**                      \* **L H L L H L**

# Accounting for Context [cont.]

## A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
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### Example

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.....  
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**A ITSL analysis for UTP** (DeSanto & Graf 2019):

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## Example

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## Example

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.....  
*ok* **L H L L** L L


\* **L H L L H L**

# Accounting for Context [cont.]

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## Example

  
*ok* L H L L L L

*\** L H L L H L

# Accounting for Context [cont.]

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- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

## Example

.....  
.....  
*ok* **L** **H** **L** **L** **L** **L**

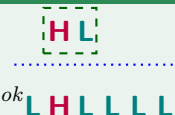
.....  
.....  
*\** **L** **H** **L** **L** **H** **L**

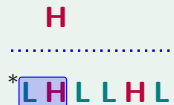
# Accounting for Context [cont.]

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- ▶ Ban: **HLH**

### Example

*ok* 



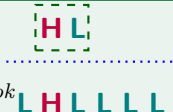


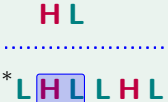
# Accounting for Context [cont.]

**A ITSL analysis for UTP** (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

## Example

*ok* 



# Accounting for Context [cont.]

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- ▶ Project every **H**; project **L** iff immediately follows **H**
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## Example

*ok* **L** **H** **L** **L** **L** **L**

The sequence **LHL** is enclosed in a dashed green box, and a dotted blue line is positioned below it.

**H** **L**

*\** **L** **H** **L** **L** **H** **L**

The sequence **LHL** is enclosed in a solid blue box, and a dotted blue line is positioned above it.

# Accounting for Context [cont.]

## A ITSL analysis for UTP (DeSanto & Graf 2019):

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### Example

*ok*

.....

**H** **L**

**L** **H** **L** **L** **L** **L**

**H** **L** **H**

.....

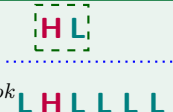
\* **L** **H** **L** **L** **H** **L**

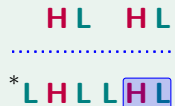
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*ok* 



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## Example

*ok*

.....

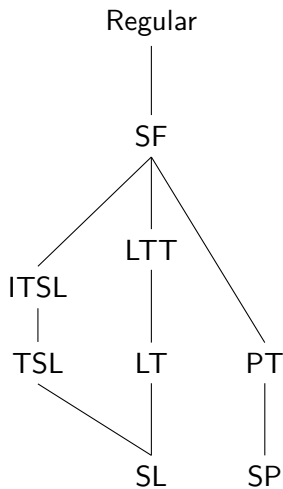
**L** **H** **L** **L** **L** **L**

*\**

.....

**L** **H** **L** **L** **H** **L**

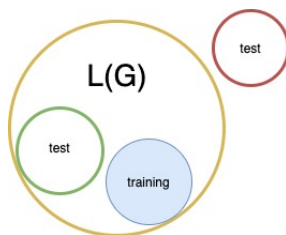
# Finer Granularity



# Outline

## 5 Limits of AGL

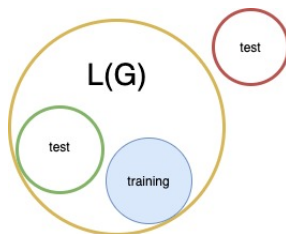
# Testing Predictions with AGL



- ▶ It is a powerful technique
- ▶ We must be careful in drawing inferences from laboratory behavior
- ▶ Importantly: Common fallacies in experimental design



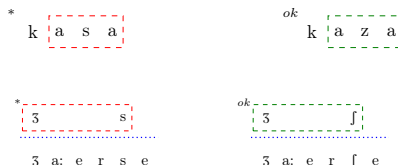
# Testing Predictions with AGL



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- ▶ We must be careful in drawing inferences from laboratory behavior
- ▶ Importantly: Common fallacies in experimental design

# The Fallacy of Generalization

- Imagine we want to test the ability to learn long-distance dependencies:



- Assuming an alphabet  $\Sigma = \{a, b, c, d, e\}$ , the training samples could look like the following:

$$L_{loc} = \{abcd, aabcd, baacd, bcaae, \dots\}$$

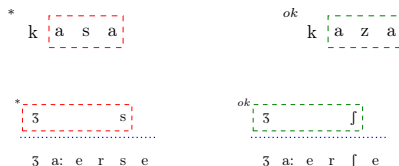
$$L_{dist} = \{abacd, bacad, bcada, bcaea, \dots\}$$

What happens if we test on stimuli with similar distances?

$$L_{test} = \{abcad, abcad, bacda, abcea, \dots\}$$

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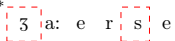
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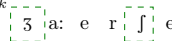
# Picking the Right Primitives

## Long-distance relations?

\*   
ʒ a: e r s e

ok   
ʒ a: e r ʃ e

\*   
ʒ a: e r s e


ok   
ʒ a: e r ʃ e

- ▶ Stimuli are often ambiguous between overlapping classes
- ▶ Distinguishing between representation requires care

# Picking the Right Primitives


Long-distance relations?

*\**



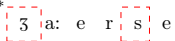
ʒ a: e r s e

*ok*



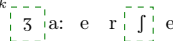
ʒ a: e r ʃ e

*\**



ʒ a: e r s e

*ok*

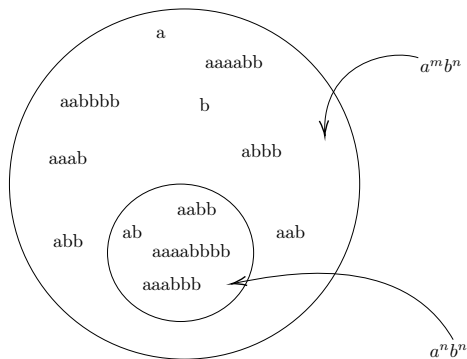


ʒ a: e r ʃ e

- ▶ Stimuli are often ambiguous between overlapping classes
- ▶ Distinguishing between representation requires care

# The Set/Subset Problem: Case 1

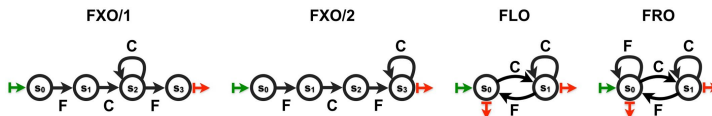
- ▶ Can participants learn  $a^n b^n$ ?
- ▶ We must beware of  $a^m b^n$



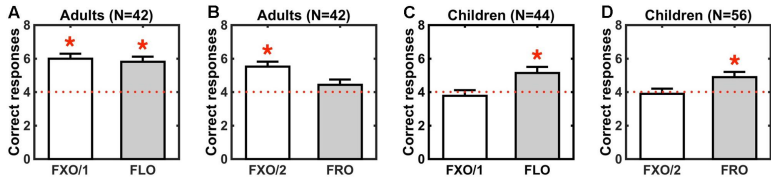
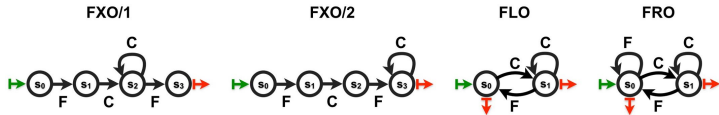
# Evaluating Contrasts

## Developmental Constraints on Learning Artificial Grammars with Fixed, Flexible and Free Word Order

 Iga Nowak<sup>1,2</sup> and  Giosuè Baggio<sup>2,3\*</sup>

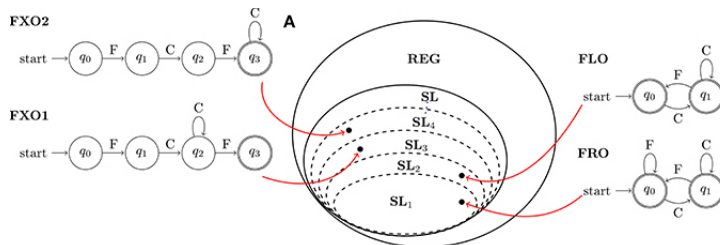
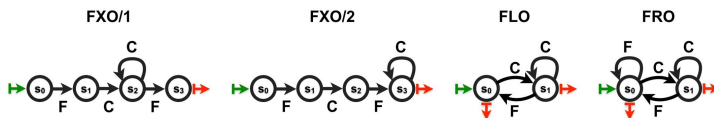


# Nowak and Baggio (2017): Results





# Complexity Measures and Other Issues (De Santo, 2017)



## The Set/Subset Problem: Case 2

- Can participants learn a truly free-word order language?

