

Learnability Insights from (and into) the TSL Neighborhood

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Theories from Data?

Theories of linguistic representations from typological/empirical observations?

The problem that we cannot deduce [...] theories from data is a limitation, or **perhaps an attribute**, of all empirical science [...] Still, one may abduce hypotheses [...] Abduction is **reasoning from observations** [...] It consists of two steps: generating candidate **hypotheses** (abduction proper), and selecting the "best" explanatory one[s] (inference to the **best explanation**).

(van Roji & Baggio 2020, pg. 9)

Theories from Data?

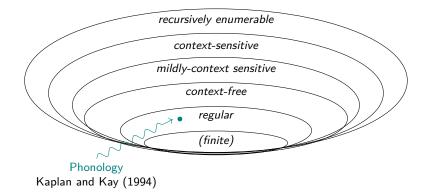
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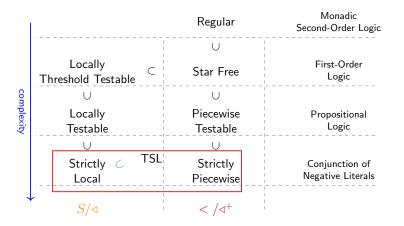
(van Roji & Baggio 2020, pg. 9)

Spoken Languages' Phonotactics as a Regular System

Stringsets can be classified according to the requirements of the grammars that generate them.

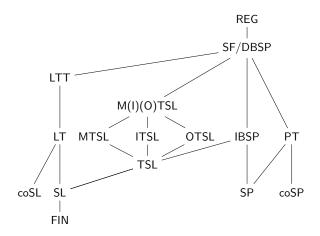


From Subclasses...¹



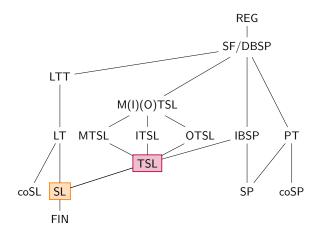
¹Subregular classes as of Heinz (2011)

... To Subclasses²



²Subregular classes as of (Lambert 2022)

... To Subclasses²



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Boundaries vs. Invariants

Descriptive characterizations focus on the **nature of the information** [...] that is needed in order to distinguish [...] a pattern.

(Rogers & Pullum 2011)

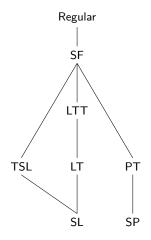
Invariants

SL: adjacency

▶ SP: precedence

► TSL: relativized adjacency

...



So let's look at subtle differences between classes!

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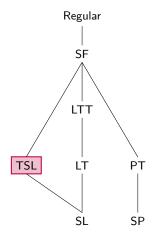
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...



So let's look at subtle differences between classes!

Unbounded Dependencies as TSL

► Ineseño Chumash Sibilant Harmony Sibilants must not disagree in anteriority. (Applegate 1972)

- (1) a. * hasxintilawaſ
 - b. * ha∫xintilawas
 - c. ha∫xintilawa∫
- What do we need to project? [+strident]
- What do we need to ban? *[+ant][-ant],*[-ant][+ant]
 I.E. *s∫, *sʒ, *z∫, *zʒ, *∫s, *ʒs, *∫z, *ʒz

Example: TSL Ineseño Chumash

S

* \$hasxintilaw[]\$

l l

 ok \$ha \mathbf{f} xintilaw \mathbf{f} \$

Unbounded Dependencies as TSL

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```
* $hasxintilaw[$
```

Ineseño Chumash: The Full Picture

Sibilant Harmony in INESEÑO CHUMASH (McMullin 2016)

1) Unbounded sibilant harmony

```
a. /k-su-\intojin/ k\intu\intojin "I darken it"
```

b. /k-su-k'ili-mekeken- \int / k \int uk'ilimekeket \int "I straighten up"

2) $/s/\rightarrow$ [ʃ] when preceding (adjacent) [t, n, l]

3) Long-distance agreement overrides local disagreement

```
a. /s-iʃt-iʃti-jep-us/ sististijepus "they show him"
```

b. /s-net-us/ snetus "he does it to him"

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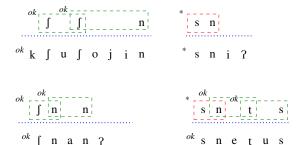
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```

Ineseño Chumash is not TSL

INESEÑO CHUMASH Sibilant Harmony (Revisited)

- anticipatory sibilant harmony [*s∫, *s∫]
- palatalization to avoid local restriction [*sn, *st, *sl]
- sibilant harmony overrides palatalization



Input-Sensitive TSL (ITSL) Languages

TSL languages are characterized by:

- ightharpoonup a 1-local projection function E_T
- ▶ strictly k-local constraints applied on T

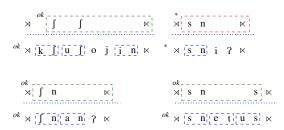
ITSL (De Santo & Graf 2019)

- ► Tier projection controlled by:
 - 1 label of segment
 - 2 n-local context
- strictly k-local constraints applied on T

An ITSL Account of Ineseño Chumash

INESEÑO CHUMASH Sibilant Harmony (Revisited)

- anticipatory sibilant harmony [*s∫, *s∫]
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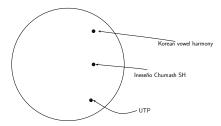


ITSL: Recap

Input-Sensitive TSL (ITSL; De Santo & Graf, 2019)

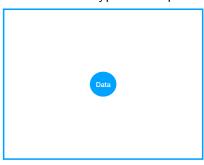
- ▶ *n*-local projection function
- \blacktriangleright strictly k-local constraints enforced on T.

- Generalization of TSL
- Covers a variety of patterns
- Gold learnable
 Efficiently learnable?



Learnability and Formal Grammars Problem:

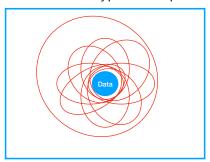
Unrestricted Hypothesis Spaces



Learnability and Formal Grammars

Problem:

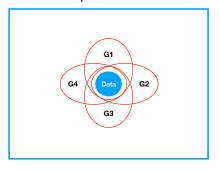
Unrestricted Hypothesis Spaces



Learnability and Formal Grammars

Solution:

Structural priors



Invariants

- ► SL: adjacency
- SP: precedence
- ► TSL: relativized adjacency
- ► ITSL: relativized adjacency + local contexts

Learning TSL and ITSL

Learning TSL_k Efficiently

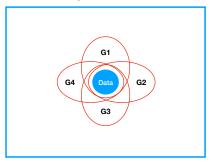
- ► Batch learning:
 - ▶ Jardine & Heinz (2016); Jardine & McMullin (2017)
 - multiple TSL: McMullin, Akenova & De Santo (2019)
- ► Incrementally: Lambert (2021)

Learning ITSL $_k^m$ Efficiently

- ► Batch learning:
 - ▶ De Santo & Aksënova (2021)
 - multiple ITSL: De Santo & Aksënova (2021), Johnson & De Santo (2023)
- ► Incrementally: Johnson & De Santo (2024)

Learning (M)ITSL Grammars ³

Structural priors



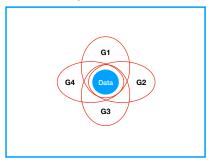
De Santo & Aksënova (2021):

- \Rightarrow Assume relativized locality!
 - tiers (but not their content)
 - contextual tier-projection
 - local tier constraints
 - characteristic sample!

³McMullin, Aksënova; De Santo (2020), De Santo & Aksënova (2021)

Learning (M)ITSL Grammars ³

Structural priors



De Santo & Aksënova (2021):

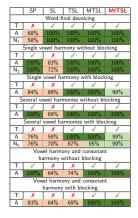
- \Rightarrow Assume relativized locality!
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Guarantees

- ▶ No a priori information on the content of tiers/constraints
- ► Guaranteed convergence in polynomial time and data

³McMullin, Aksënova; De Santo (2020), De Santo & Aksënova (2021)

Evaluating Convergence in Real World Scenarios⁴⁵





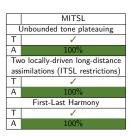
Get the Code!

⁴(T)heoretical expectations and performance of 5 subregular learners on (A)rtificial and simplified (N)atural language input data-sets. N₁: German; N₁: Finnish; N₁: Turkish.

Aksënova (2020), De Santo & Aksënova (2021), Johnson & De Santo (2023)

Evaluating Convergence in Real World Scenarios⁴⁵

	SP	SL	TSL	MTSL	MITSL
	Word-final devoicing				
T	X	✓	✓	1	✓
Α	68%	100%	100%	100%	100%
N_1	58%	100%	100%	100%	100%
	Single vo	owel harr	mony wit	thout blo	cking
T	✓	X	✓	1	✓
Α	100%	83%	100%	100%	100%
N_2	100%	72%	100%	100%	100%
	Single	vowel ha	armony v	vith block	king
Т	X	X	✓	1	✓
Α	84%	89%	100%	100%	99%
	everal vo	owel harr	monies w	rithout bl	ocking
Т	✓	X	✓	✓	✓
Α	100%	69%	100%	100%	100%
	Several	vowel ha	rmonies	with bloo	cking
Т	X	X	✓	1	✓
Α	76%	59%	100%	100%	99%
N_3	76%	70%	67%	95%	99%
				consonan	t
harmony without blocking					
Т	1	X	X	>	1
Α	100%	64%	74%	100%	100%
Vowel harmony and consonant					
harmony with blocking					
Т	Х	X	X	1	1
A	83%	64%	69%	100%	100%





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				consonan	t
harmony without blocking					
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Vowel harmony and consonant					
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Т	Х	X	X	1	1
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Attested vs. Unattested (?) Patterns

Attested: Unbounded Sibilant Harmony

► Every sibilant needs to harmonize

```
s ʃ
```

*\$ha**s**xintilaw∫\$

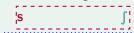
ok\$ha∫xintilaw∫\$

Unattested (?): First-Last Harmony

► Harmony only holds between initial and final segments

```
s ʃ
```

^{ok}\$ha**s**xintilaw∫\$



*\$satxintilawſ\$

Learnable vs. Unlearnable (?) Patterns





Learnable vs. Unlearnable Harmony Patterns

Regine Lai

Posted Online July 09, 2015 https://doi.org/10.1162/LING a 00188

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Keywords: phonotactics, learnability, computational phonology, formal theory, typology, dependencies

Lai (2015): Stimuli & Predictions

 Table 4

 Types of training items used in the Sibilant Harmony, First-Last, and control conditions. Vowels are omitted. (No training took place in the control condition.)

	Conditions		
Sibilant tier	Sibilant Harmony	First-Last	
[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]	
$[[1, \dots, [1], \dots]]$	[ʃkʃʃ] [ʃʃkʃ]	[ʃkʃʃ] [ʃʃkʃ]	
$[s\ldots \int \ldots s]$	None	[s k ∫ s] [s ∫ k s]	
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]	

Lai (2015): Stimuli & Predictions

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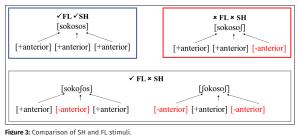
	Conditions		
Sibilant tier	Sibilant Harmony	First-Last	
[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]	
$[[\ldots]\ldots[]$	[ʃkʃʃ] [ʃʃkʃ]	[ʃkʃʃ] [ʃʃkʃ]	
$[s\dots f\dots s]$	None	$ [s \dots k \dots f \dots s] $ $ [s \dots f \dots k \dots s] $	
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]	

Table 5

Predicted preferences for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

	Pairs				
Conditions	FL/*SH vs. *FL/*SH (e.g., [s f s] vs. [s s [])	FL/SH vs. *FL/*SH (e.g., [s s s] vs. [s s]])	FL/SH vs. FL/*SH (e.g., [s s s] vs. [s ∫ s])		
SH FL Control	No preference $[s \dots f \dots s] > [s \dots s \dots f]$ No preference	$ [s \dots s \dots s] > [s \dots s \dots \int] $ $ [s \dots s \dots s] > [s \dots s \dots \int] $ No preference	[sss] > [sfs] No preference No preference		

Lai (2015): Results⁶



rigure 3: Comparison of SH and FL Sumuli.

TL/DR

- ▶ SH participants perform in line with a SH grammar
- ► FL not leaned
- but FL participants perform in line with a SH-like grammar

⁶See Avcu & Hestvik (2020) for a partial replication.

Lai (2015) vs MITSLIA

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[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]	
$[\mathcal{I} \cdots \mathcal{I} \cdots \mathcal{I}]$	[ʃkʃʃ] [ʃʃkʃ]	[ʃkʃʃ] [ʃʃkʃ]	
$[s \dots f \dots s]$	None	$\begin{bmatrix} s \dots k \dots \int \dots s \end{bmatrix}$ $\begin{bmatrix} s \dots \int \dots k \dots s \end{bmatrix}$	
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]	

What if we train the (M)ITSL batch algorithm on Lai's stimuli?⁷

- ▶ is the SH training a characteristic sample?
- is the FL training a characteristic sample?



⁷Johnson & De Santo (2023); De Santo, Johnson, Aksënova (in prep.)

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Sibilant tier	Sibilant Harmony	First-Last	
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[[]]]	$ \begin{bmatrix} \int \dots k \dots \int \dots \int \end{bmatrix} $ $ [\int \dots \int \dots k \dots \int] $	[ʃkʃʃ] [ʃʃkʃ]	
[s ∫ s]	None	[s k ∫ s] [s ∫ k s]	
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]	

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$[s\ldots \smallint \ldots s]$	None	[s k ∫ s] [s ∫ k s]	
[ʃsʃ]	None	$ \begin{bmatrix} \int \dots k \dots s \dots \int \end{bmatrix} $ $ \left[\int \dots s \dots k \dots \int \right] $	

What if we train the (M)ITSL batch algorithm on Lai's stimuli?⁷

- ▶ is the SH training a characteristic sample? No!
- is the FL training a characteristic sample? No!
- what if: information about natural classes?

E.g. vowels

⁷Johnson & De Santo (2023); De Santo, Johnson, Aksënova (in prep.)



Lai (2015) vs. MITSLIA: Masked SH Training

Performance of ITSL Batch Learner:

- trained on Lai (2015)'s SH input
 - \rightarrow 40 words, masking vowels;
- tested on acceptance of 96 individual test strings.

```
Evaluating g sh
    What should this accept? (highlighted yellow)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   svkvSvs SvkvSvs svkvsvs svkv
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What does this accept? (highlighted magenta)
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Lai (2015) vs. MITSLIA: Masked FL Training

Performance of ITSL Batch Learner:

- trained on Lai (2015)'s FL input
 - ightarrow 40 words, masking vowels;
- tested on acceptance of 96 individual test strings.

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Evaluating q fl
What should this accept? (highlighted vellow)
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Lai (2015): Intensive FL Training

Table 4Types of training items used in the Sibilant Harmony, First-Last, and control conditions. Vowels are omitted. (No training took place in the control condition.)

	Conditions	
Sibilant tier	Sibilant Harmony	First-Last
[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]
$[\mathcal{S} \cdots \mathcal{S} \cdots \mathcal{S}]$	$ \begin{bmatrix} \int \dots k \dots \int \dots \int \end{bmatrix} $ $ [\int \dots \int \dots k \dots \int] $	[ʃkʃʃ] [ʃʃkʃ]
$[s \dots f \dots s]$	None	$\begin{bmatrix} s \dots k \dots \int \dots s \end{bmatrix}$ $\begin{bmatrix} s \dots \int \dots k \dots s \end{bmatrix}$
[ʃsʃ]	None	$ \begin{bmatrix} \int \dots k \dots s \dots \int \end{bmatrix} $ $ [\int \dots s \dots k \dots \int] $

Lai (2015): Intensive FL Training

Table 4Types of training items used in the Sibilant Harmony, First-Last, and control conditions. Vowels are omitted. (No training took place in the control condition.)

	Conditions	
Sibilant tier	Sibilant Harmony	First-Last Intensive
[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]
$[[] \cdots] \cdots []$	$ \begin{bmatrix} \int \dots k \dots \int \dots \int \end{bmatrix} $ $ [\int \dots \int \dots k \dots \int] $	[ʃkjʃ] [gʃkʃ]
$[s \dots f \dots s]$	None	$\begin{bmatrix} s \dots k \dots \int \dots s \end{bmatrix}$ $\begin{bmatrix} s \dots \int \dots k \dots s \end{bmatrix}$
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]

Lai (2015): Intensive FL Training

Table 4

Types of training items used in the Sibilant Harmony, First-Last, and control conditions. Vowels are omitted. (No training took place in the control condition.)

	Conditions	
Sibilant tier	Sibilant Harmony	First-Last Intensive
[s s s]	[s k s s] [s s k s]	[s k s s] [s s k s]
$[\dots] \dots []$	$ \begin{bmatrix} \int \dots k \dots \int \dots \int \end{bmatrix} $ $ [\int \dots \int \dots k \dots \int] $	[ʃkjʃ] [ʃʃkʃ]
$[s \dots \int \dots s]$	None	$\begin{bmatrix} s \dots k \dots \int \dots s \end{bmatrix}$ $\begin{bmatrix} s \dots \int \dots k \dots s \end{bmatrix}$
[ʃsʃ]	None	[ʃksʃ] [ʃskʃ]

TL/DR

- ► FL not leaned but IFL participants exhibit regularities...
- ... in line with a sibilant disharmony rule

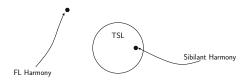
Lai (2015) vs. MITSLIA: Masked IFL Training

Performance of ITSL Batch Learner:

trained on Lai (2015)'s Intensive FL input

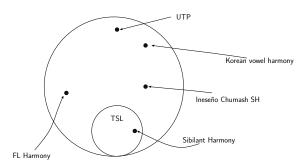
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Does the grammar make the same judgement as the fl acceptor? 66.67% svkvSvs SvkvSvs svkvsvs svkv
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Subregularity, Typology, & AGL in the TSL Neighborhood



- ► Comparing behavior within classes is essential
- ► Different characterizations and/or non-FLT explanations (Garret & Johnson 2011, Endress et al. 2005; Endress & Mehler 2010)
- ► How does this tie to cognitive resources (e.g. WM)? (Baddeley 2000, Pierce et al. 2017)

Subregularity, Typology, & AGL in the TSL Neighborhood



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- ▶ Different characterizations and/or non-FLT explanations (Garret & Johnson 2011, Endress et al. 2005; Endress & Mehler 2010)
- ► How does this tie to cognitive resources (e.g. WM)? (Baddeley 2000, Pierce et al. 2017)

Conclusion

FLT & Typology: Representational primitives!

- Do we look for restrictive theories?
- ▶ YES! But driven by invariants, not boundaries

AGL: Probe which primitives humans are sensitive to!

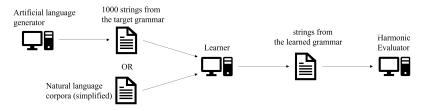
- Compare between classes and within classes!
- ► Formal Learners provide insights/ground truths
- Not just about complexity: TSL vs. SP?
- ► AGL Design: Not just FLT & Typology!
 - ► Working memory, attention, input source, etc.
 - Look at psycholinguists/laboratory phonologists/etc.

Thank You!

Special thanks to Jon Rawski, Caleb Belth, Yang Wang, Alëna Aksënova, Jacob Johnson, and members of the Utah Comp Ling Working Group for discussions on several aspects of this work!

Appendix

Evaluation⁸



- ► TSL and ITSL mplemented in Python 3 following requirements of SigmaPie
- Artificial datasets exemplifying different subregular classes
- 3 simplified natural language corpora (German, Finnish, Turkish)
- Proportion of first 5000 strings accepted by the learned grammar also accepted by the target grammar
- ► Learning + evaluation iterated 10 times

⁸ Aksenova (2020); Johnson & De Santo (2023, 2024)

Online Learner: Evaluation

	TSL	ITSL	
	Word-final devoicing		
Т	1	✓	
Α	100%	100%	
N_{G}	100%	100%	
Single vowel harmony without blocking			
Т	1	✓	
Α	100%	100%	
N _F	100%	100%	
S	ingle vov	vel harmony with blocking	
Т	1	✓	
Α	100%	100%	
Seve	ral vowe	I harmonies without blocking	
Т	1	✓	
Α	100%	100%	
Several vowel harmonies with blocking			
Т	1	✓	
Α	100%	100%	
N _T	100%	100%	

Online Learner: Evaluation

	TSL	ITSL
Word-final devoicing		
Т	1	✓
Α	100%	100%
N _G	100%	100%
Single vowel harmony without blocking		
Т	1	✓
Α	100%	100%
N _F	100%	100%
S	ingle vov	vel harmony with blocking
Т	1	✓
Α	100%	100%
Several vowel harmonies without blocking		
Т	1	✓
Α	100%	100%
Several vowel harmonies with blocking		
Т	1	✓
Α	100%	100%
N _T	100%	100%

	TSL	ITSL		
Unbounded tone plateauing				
Т	Х	1		
Α	9.97% (0.51%)	100%		
	First-Last Assimilation			
Т	X	✓		
Α	50.02%	100%		
	Locally-driven long-distance			
assimilation (ITSL restriction)				
Т	X	✓		
Α	94.88% (0.15%)	100%		

Why Care?

Regularity then provides a well-defined proposed computational universal that is sufficiently expressive while ruling out a great many non-phonological patterns.

(Chandlee 2024)

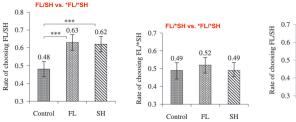
The larger goal of this work is to formally delimit the boundary between possible and impossible grammatical patterns, as evidenced by attested and unattested language patterns.

(McCollum et al.)

The Weak Subregular Hypothesis (Heinz, 2018:155) restricts phonotactic generalizations to strings over tiers [...] While it is not difficult to find apparent counterexamples to the Weak Subregular Hypothesis cross-linguistically, they are often explained by invoking standard phonological structures.

(Lamont, u.r.)

Lai (2015): Results⁹



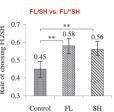


Table 6
Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

	Pairs			
Conditions	FL/*SH vs. *FL/*SH (e.g., $[s \dots f \dots s]$ vs. $[s \dots s \dots f]$) Rate of FL/*SH	FL/SH vs. *FL/*SH (e.g., [s s s] vs. [s s ʃ]) Rate of FL/SH	FL/SH vs. FL/*SH (e.g., [s s s] vs. [s ∫ s]) Rate of FL/SH	
SH FL	~ Control > Control	> Control > Control	> Control ~ Control	

⁹See Avcu & Hestevik (2020) for a partial replication.

Lai (2015): Results (Part 2)

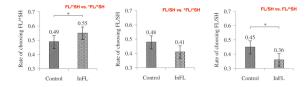


Table 6
Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

Conditions	Pairs			
	FL/*SH vs. *FL/*SH (e.g., $[s \dots f \dots s]$ vs. $[s \dots s \dots f]$) Rate of FL/*SH	FL/SH vs. *FL/*SH (e.g., [s s s] vs. [s s ʃ]) Rate of FL/SH	FL/SH vs. FL/*SH (e.g., [s s s] vs. [s ∫ s]) Rate of FL/SH	
SH FL	~ Control > Control	> Control > Control	> Control ~ Control	