Evaluating Subregular Distinctions in the Complexity of Generalized Quantifiers

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The Talk in a Nutshell

Generalized Quantifiers and Semantic Complexity

Semantic automata (SA) as a model of quantifiers'verification

- insights into quantifiers' interpretation
- link between formal language theory and model theory

In This Talk: Giving up the SA perspective

- But: formal language theory is richer that automata theory
- Coming back to formal language theory
  $\rightarrow$ subregular hierarchy & quantifier languages

Consequences

- complexity independent of the recognition mechanism
- cross-domain parallels, cognitive predictions and new experimental paradigms!

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Outline

1 Semantic Automata

2 Generalized Quantifiers & Subregular Languages

3 Psycholinguistic Predictions

4 Conclusions

Generalized Quantifiers

Generalized quantifier $\mathbf{Q}(\mathbf{A}, \mathbf{B})$:

- two sets **A** and **B** as arguments
- returns truth value $(0, 1)$

Example

(1) Every student cheated.

- $\mathbf{every}(\mathbf{A}, \mathbf{B}) = 1$ iff $\mathbf{A} \subseteq \mathbf{B}$
- **student**: John, Mary, Sue
- **cheat**: John, Mary
- $\mathbf{student} \not\subseteq \mathbf{cheat} \Rightarrow \mathbf{every}(\mathbf{student}, \mathbf{cheat}) = 0$
- "Every student cheated" is false.

Binary Strings

- The language of **A** is the set of all permutations of **A**.

Example

student	John, Mary, Sue
$L($**student**$)$	John Mary Sue, John Sue Mary
	Mary John Sue, Mary Sue John
	Sue John Mary, Sue Mary John

- Now replace every **a** $\in$ **A** by a truth value:
  1 if **a** $\in$ **B**
  0 if **a** $\notin$ **B**
- The result is the **binary string language** of **A** under **B**.

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Quantifier Languages (van Benthem 1986)

- We can associate each quantifier Q with a language in $\{0,1\}^*$
  $\Rightarrow$ Q accepts only binary strings of specific shape
- This is its **quantifier language**.

Example: *every*

- **every**(**A**, **B**) holds iff **A** $\subseteq$ **B**
- So every element of **A** must be mapped to 1.
- $L(\textbf{every}) = \{1\}^*$

Example: *some*

- **some**(**A**, **B**) holds iff **A** $\cap$ **B** $\neq \emptyset$
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- $L(\textbf{some}) = \{0,1\}^* 1 \{0,1\}^*$

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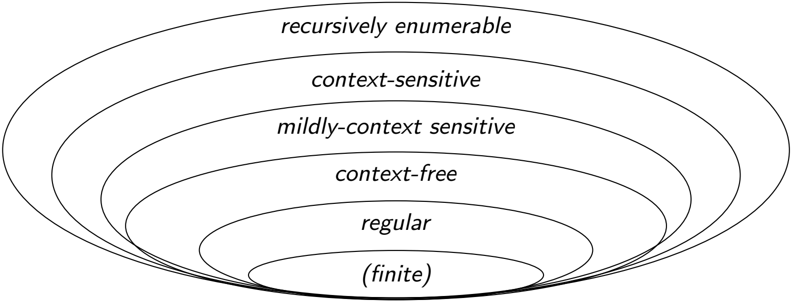
Languages (stringsets) can be classified according to the complexity of the grammars that generate them.



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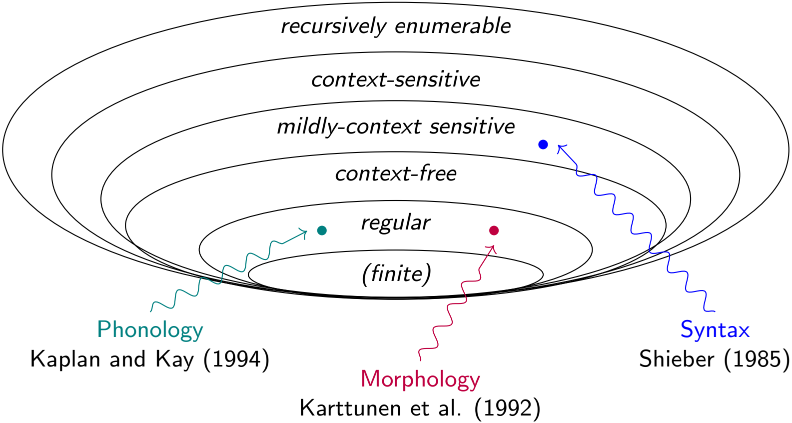
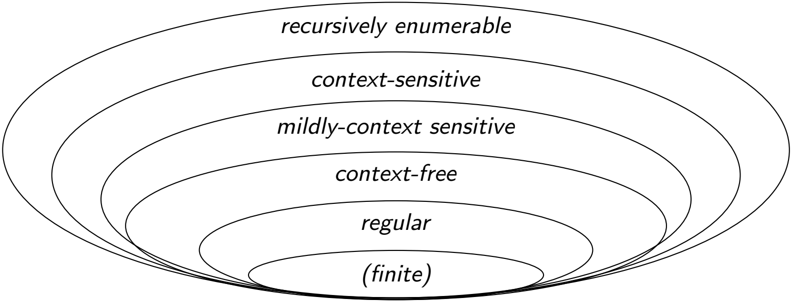


Chomsky Hierarchy and Automata Theory



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We can rank quantifiers based on their quantifier languages and the complexity of the machine needed to recognize them.

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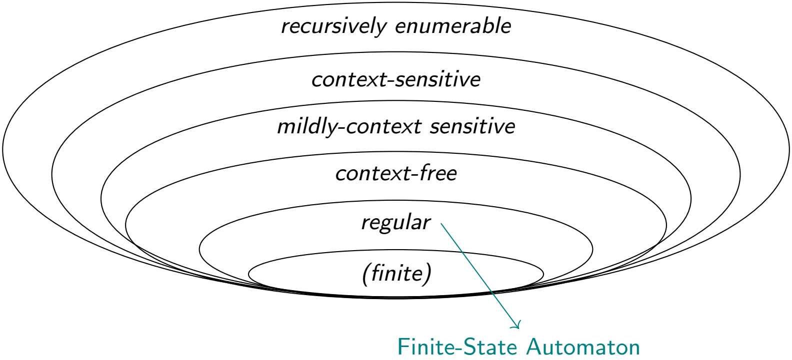

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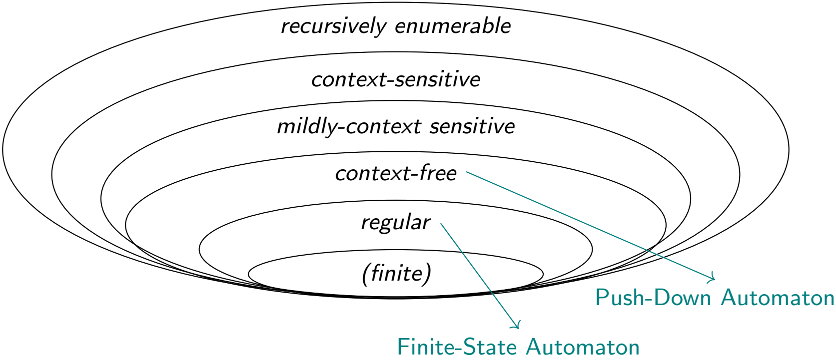

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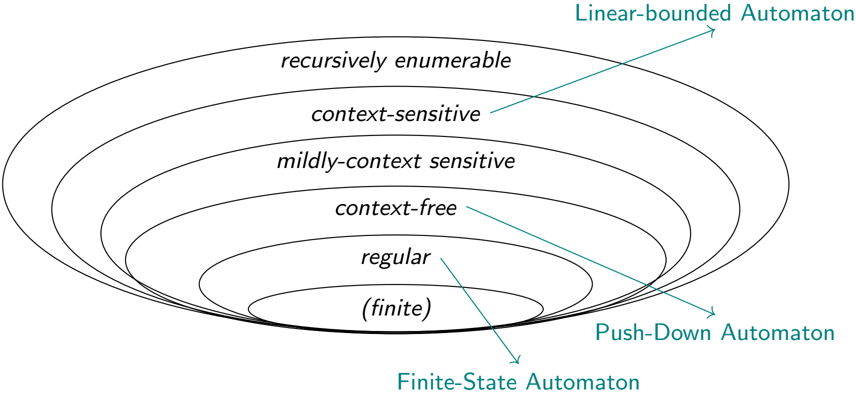

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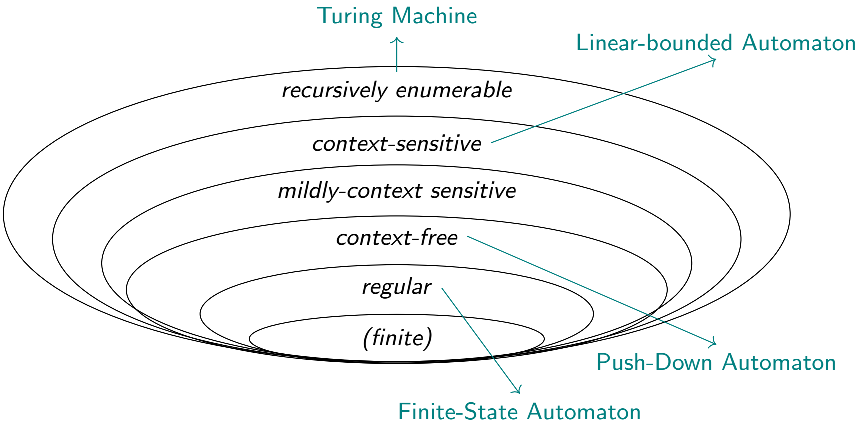

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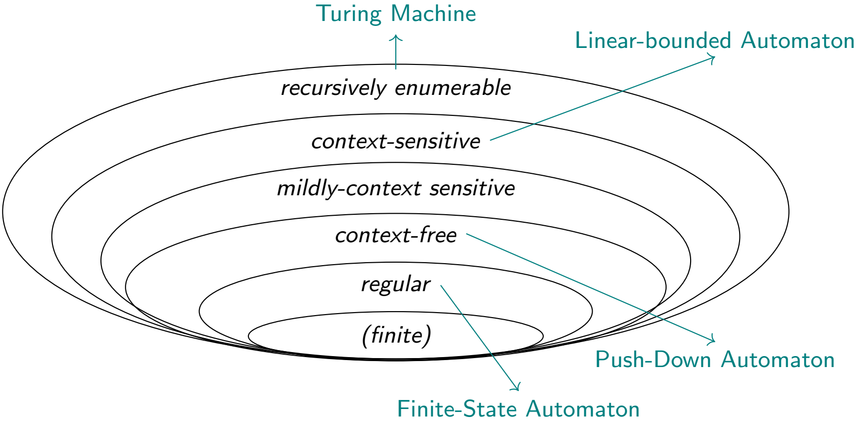

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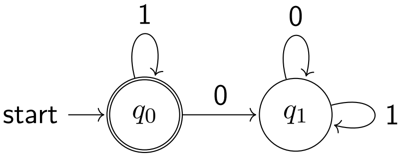

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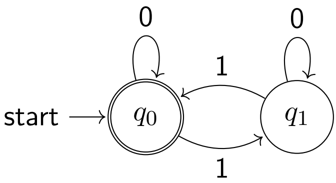
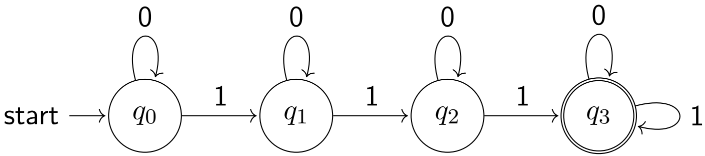
Other FSA-recognizable quantifiers

- Parity quantifiers: **An even number**



- Cardinal quantifiers: **At least 3**



Proportional Quantifiers

- **most**(**A**, **B**) holds iff $|\mathbf{A} \cap \mathbf{B}| > |\mathbf{A} - \mathbf{B}|$
- $L_{most} := \{w \in \{0,1\}^* : |1|_w > |0|_w\}$
- There is no finite automaton recognizing this language.
- We need internal memory.
  $\Rightarrow$ **push-down automata**: two states + a stack

A Hierarchy of Quantifiers' Complexity

FSA **PDA**

$\boxed{\{\textit{All, Some, Even, Odd, At least n, At most n}\}} < \boxed{\{\textit{Less than half, More than half, Most}\}}$

Are these all of equivalent complexity?

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Let's Look at the Automata One More Time

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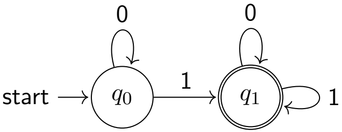

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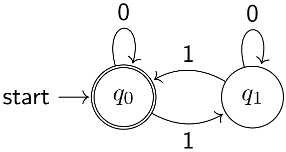

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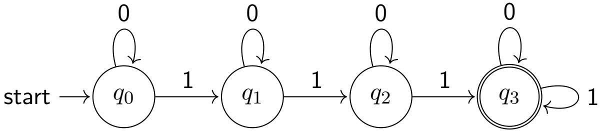

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Are these all of equivalent complexity? (Szymanik 2016)

- Cyclic vs acyclic automata
- The number of states matters
- But: Complexity = succinctness of automata?

Reminder

It's all grounded in quantifier languages

- FSA recognizable quantifiers → Regular quantifier languages

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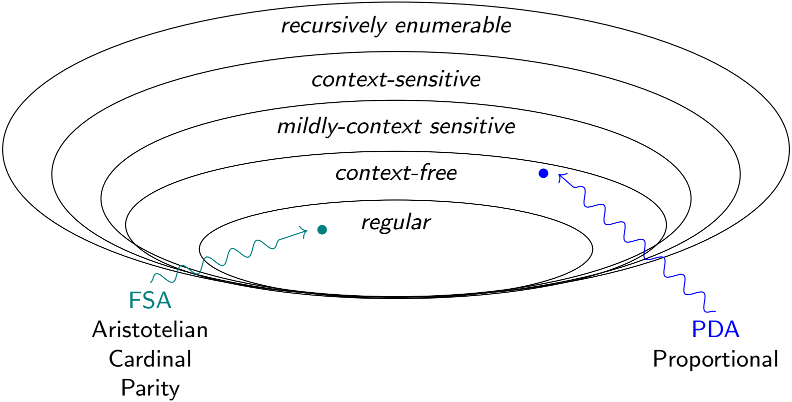

The Subregular Hierarchy

Often Forgotten:

- hierarchy of subregular languages (McNaughton&Papert 1971), (Rogers et al. 2010)

A Richness of Results

- Phonology is subregular (Heinz&Idsardi 2013, Heinz 2015)
- Morphotactics (and Morphology?) is subregular (Aksënova et al. 2016, Chandlee 2016, Aksënova&De Santo 2017)
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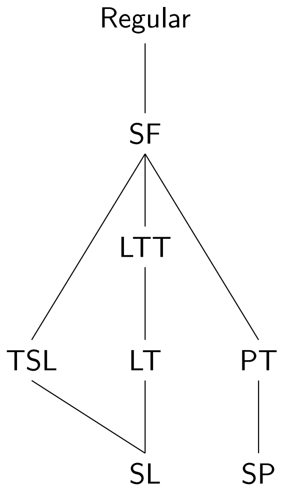

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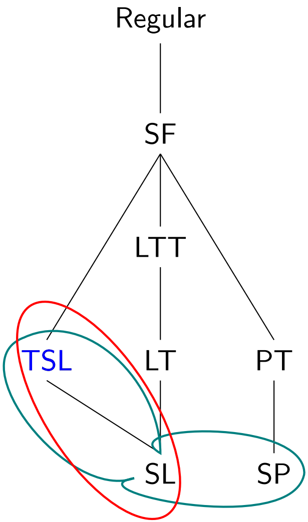

Strictly Local and Tier-based Strictly Local

Strictly Local (SL)

- SL_k grammars are lists of forbidden k-grams;

Tier-based Strictly Local (TSL)

- TSL is a minimal extension of SL, inspired by phonological tiers;
  - define a subset T of the string alphabet;
  - a grammar is a list of strictly k-local constraints over T

Subregular Quantifiers: *Every is SL*

Reminder: *Every*

- **every**(**A**, **B**) holds iff **A** $\subseteq$ **B**
- $L(\textbf{every}) = \{1\}^*$
- Eg. *Every student cheated.*

False

student	John, Mary, Sue
cheat	John, Mary
binary strings	110, 101, 011
grammar	*0

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- $L(\textbf{some}) = \{0,1\}^* \, 1 \, \{0,1\}^*$
- Eg. *Some student cheated.*

False

student	John, Mary, Sue
cheat	
binary strings	000
grammar	^{*0}

$\rtimes$ 0 0 $\boxed{0}$ $\ltimes$

True

student	John, Mary, Sue
cheat	John
binary strings	100,010,001
grammar	^{*0}

$\rtimes$ 0 0 1 $\ltimes$

Subregular Quantifiers: *Some* is SL?

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$\rtimes \;\; 0 \;\; 0^n \;\; 0 \;\; \ltimes$

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T $\boxed{\rtimes \quad 1}$ $\boxed{1 \quad \ltimes}$

$\rtimes \;\; 0^n \;\; 1 \;\; 0^n \;\; \ltimes$

Parity Quantifiers?

An even number

- **An even number**(**A**, **B**) holds iff |**A** ∩ **B**| $\geq 2n$, with $n > 0$
- $L($**even**$) = \{w \in 0, 1^* s.t. |1|_w \geq 2n$, with $n > 0\}$

Is $L($**even**$)$ a TSL language?

F 1 1 1 0 0 T 1 1 1 1 0 F 1 1 1 1 1

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F 1 1 1 0 0   T 1 1 1 1 0   F 1 1 1 1 1

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1 1 1	1 1 1 1	1 1 1 1 1
F 1 1 1 0 0	T 1 1 1 1 0	F 1 1 1 1 1

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F $\boxed{1\ 1\ 1}$ — F $\boxed{1\ 1\ 1}$ 1 — F $\boxed{1\ 1\ 1}$ 1 1

F 1 1 1 0 0 — T 1 1 1 1 0 — F 1 1 1 1 1

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F 1 1 1 T 1 1 1 1 T 1 1 1 1 1

F 1 1 1 0 0 T 1 1 1 1 0 F 1 1 1 1 1

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F 1 1 1	T 1 1 1 1	F 1 1 1 1 1
F 1 1 1 0 0	T 1 1 1 1 0	F 1 1 1 1 1

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Is $L(\textbf{even})$ a TSL language?

F	T	F
$\boxed{1\ 1\ 1}$	$\boxed{1\ 1\ 1\ 1}$	$\boxed{1\ 1\ 1\ 1\ 1}$
F 1 1 1 0 0	T 1 1 1 1 0	F 1 1 1 1 1

Since n is arbitrary, there is **no general TSL grammar** that can generate $L(\textbf{even})$.

Characterization of Quantifier Languages: Summary

Language	Constraint	Complexity	Subregular Grammar
every	$\|0\|_w = 0$	SL-1	**S** $:= \{\neg 0\}$
no	$\|1\|_w = 0$	SL-1	**S** $:= \{\neg 1\}$
some	$\|1\|_w \geq 1$	TSL-2	**T** $:= \{1\}$, **S** $:= \{\neg \rtimes \ltimes\}$
not all	$\|0\|_w \geq 1$	TSL-2	**T** $:= \{0\}$, **S** $:= \{\neg \rtimes \ltimes\}$
(at least) n	$\|1\|_w \geq n$	TSL-$(n+1)$	**T** $:= \{1\}$, **S** $:= \left\{\neg \rtimes 1^k \ltimes\right\}_{k \leq n}$
(at most) n	$\|1\|_w \leq n$	TSL-$(n+1)$	**T** $:= \{1\}$, **S** $:= \left\{\neg 1^{k+1}\right\}$
all but n	$\|0\|_w = n$	TSL-$(n+1)$	**T** $:= \{0\}$, **S** $:= \left\{\neg 0^{n+1}, \neg \rtimes 0^k \ltimes\right\}_{k \leq n}$
even number	$\|1\|_w = 2n,\ n \geq 0$	regular	**impossible**
most	$\|1\|_w \geq \|0\|_w$	context-free	**impossible**

A Complexity Hierarchy (Revisited)

- Semantic Automata predictions

FSA	PDA
$\{$*All, Some*$\} < \{$*Even, Odd*$\} < \{$*At least n, At most n*$\}$	$< \{$*Less than half, More than half, Most*$\}$

- Subregular characterization predictions

SL	TSL	REG	CF
$\{$*All*$\}$	$< \{$*Some, At least n, At most n*$\}$	$< \{$*Even, Odd*$\}$	$< \{$*Less than half, More than half, Most*$\}$

Automata vs Quantifier Languages

- cardinal $<$ parity;
- complexity independent of the specific recognition machine
- **what's the cognitive reality of these predictions?**

A Complexity Hierarchy (Revisited)

- Semantic Automata predictions

FSA				PDA
{*All*, *Some*}	< {*Even*, *Odd*}	< {*At least n*, *At most n*}	<	{*Less than half*, *More than half*, *Most*}

- Subregular characterization predictions

SL		TSL		REG		CF
{*All*}	<	{*Some*, *At least n*, *At most n*}	<	{*Even*, *Odd*}	<	{*Less than half*, *More than half*, *Most*}

Automata vs Quantifier Languages

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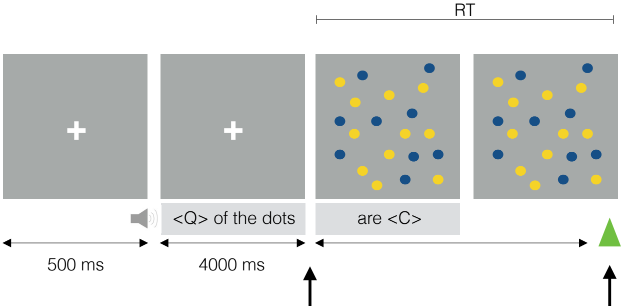
- **what's the cognitive reality of these predictions?**

Formal Complexity and Cognition

- FO-quantifiers vs higher order quantifiers
  - neuroimaging (McMillan et al. 2005, Clark & Grossman 2007)
  - patient literature (McMillan et al. 2009, Troiani et al 2009,)
- Psycholinguistic evidence for semantic automata
  - many behavioral findings (Szymanik & Zajenkowsky 2009, 2010, Steinert-Threlkeld & Icard 2013, i.a.)
  - for a survey: Szymanik (2016)
- Subregular hierarchy and cognition
  - general discussion(Rogers et al. 2013)
  - animal vs human cognition (Pulum & Rogers 2006, Rogers & Pullum 2011)
  - learnability and acquisition (Lai 2015, Avcu 2017)

Testing the Subregular Predictions



Initial encoding **Verification**

- McMillan et al. (2005)

Disentangling encoding from verification

- ERP → upcoming (De Santo et al, SNL2017), MEG, ...
- Pupil size ← **ongoing...**

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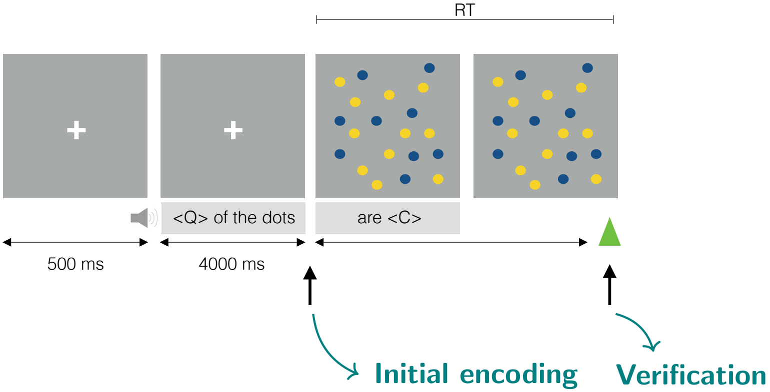

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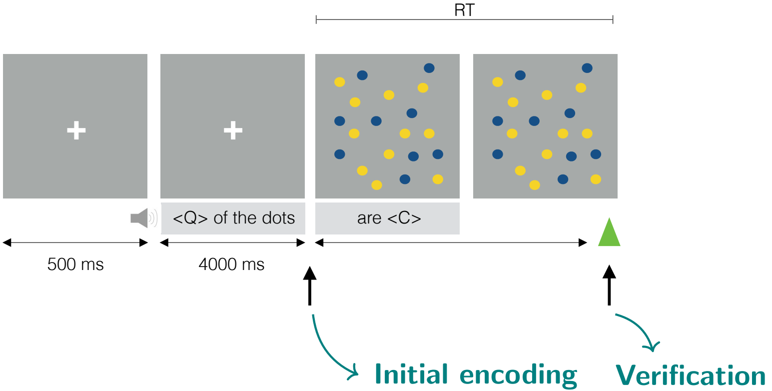

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Conclusions

Tracing Back our Steps

- SA as a plausible model of quantifier complexity
- Refined by looking at weaker classes in the Chomsky hierarchy
  $\Rightarrow$ **subregular characterization of generalized quantifiers**

Outcomes & Future Work

- Computational complexity and cognition
  - precise, testable predictions about cognitive resources
  - strong, cross-domain linking hypothesis
- Support for cross-domain subregular generalizations
  - typological predictions (Graf 2017)
  - insights on learnability/acquisition
- New theoretical questions
  - e.g. permutation closure & subregular languages?

Conclusions

Tracing Back our Steps

- SA as a plausible model of quantifier complexity
- Refined by looking at weaker classes in the Chomsky hierarchy
  $\Rightarrow$ **subregular characterization of generalized quantifiers**

Outcomes & Future Work

- Computational complexity and cognition
  - precise, testable predictions about cognitive resources
  - strong, cross-domain linking hypothesis
- Support for cross-domain subregular generalizations
  - typological predictions (Graf 2017)
  - insights on learnability/acquisition
- New theoretical questions
  - e.g. permutation closure & subregular languages?

An algorithm is likely to be understood more readily by understanding **the nature of the problem** *being solved than by examining the mechanism (and the hardware) in which it is embodied.*

(Marr 1983, p.27)

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Appendix

Logical Definability of Subregular Classes

	Regular	Monadic Second-Order Logic
	$\cup$	
Locally Threshold Testable $\subset$	Star Free	First-Order Logic
$\cup$	$\cup$	
Locally Testable	Piecewise Testable	Propositional Logic
$\cup$	$\cup$	
Strictly Local $\subset$ TSL	Strictly Piecewise	Conjunction of Negative Literals
$S/\triangleleft$	$< /\triangleleft^{+}$	

Strictly Local: Example

Word-Final Devoicing is SL

- SL grammars are lists of forbidden n-grams;
- Word-Final Devoicing: voiced segments at the end of a word are forbidden.

Word-Final Devoicing: German Example

- Grammar $S := \{{}^*z\ltimes, {}^*v\ltimes, {}^*d\ltimes\}$
- $\{\rtimes, \ltimes\} \rightarrow$ left and right word edge

$$^* \rtimes \; r \; a \; d \; \ltimes$$

$$\rtimes \; r \; a \; t \; \ltimes$$

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$$^{*}\quad \rtimes \quad r \quad a \quad \boxed{d \quad \ltimes}$$

$$^{ok}\quad \rtimes \quad r \quad a \quad \boxed{t \quad \ltimes}$$

Tier-based Strictly Local (Heinz et al. 2011)

- TSL is a minimal expansion of SL
- Inspired by phonological tiers

TSL Grammars

- a projection function $E : \Sigma \to T \cup \lambda$, with $T \subseteq \Sigma$
- strictly local constraints over T

TSL Example: Sibilant harmony in Aari

- If multiple sibilants {s, z, ʒ, ʃ} occur in the same word, they must all be voiceless {s, ʃ} or voiced {z, ʒ} .

Grammar

- T := {s, z, ʒ, ʃ}
- S := {*ʒs, *sʒ, *sʃ, *ʃs}

* ʒ aː e r s e

^{ok} ʒ aː e r ʃ e

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ʒ

T: sibilant harmony

* ʒ a: e r s e

ok ʒ a: e r ʃ e

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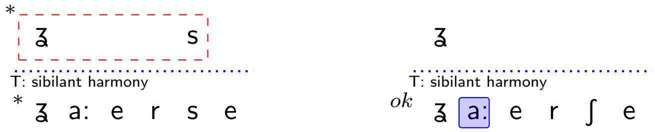

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