Mathematical Linguistics & Typological Complexity

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SIGTYP Lecture Series
Nov 19, 2021

(Some) Big Questions

- Are there **laws** that govern linguistic knowledge?
- **Why** are those the laws?
- Do they relate to **typological gaps**, i.e. logically possible patterns we don't (seem to) find?
- What can we infer about **human learning processes**?

Cross-disciplinarity for the win

- Stand on the shoulders of giants.
- Cross-fertilization and multiple explanatory levels.
- Yields new generalizations and data.

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Outline

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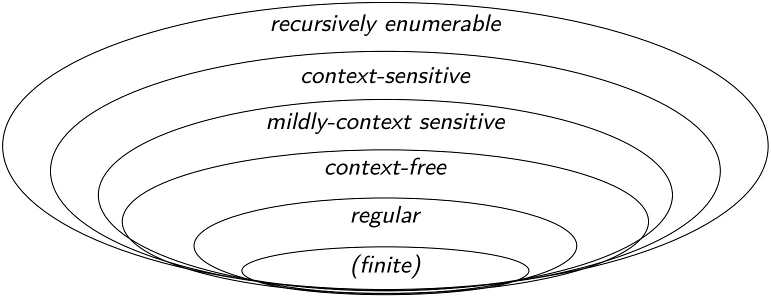
Languages (stringsets) can be classified according to the complexity of the grammars that generate them.



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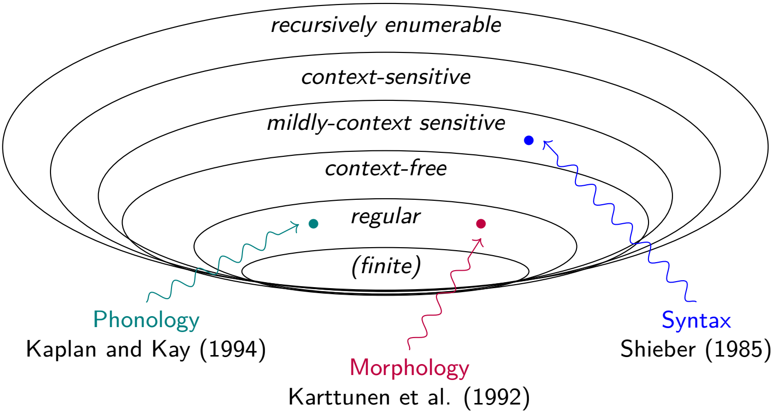
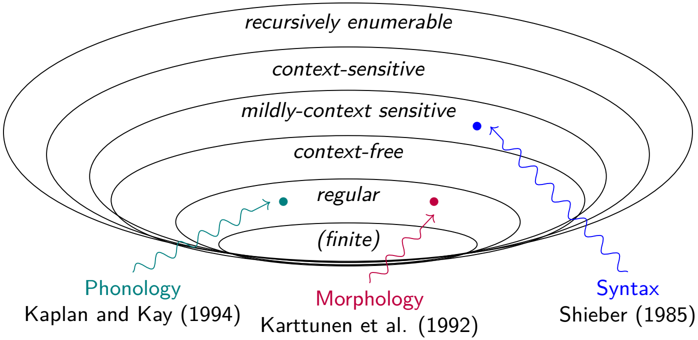


Precise Theories ⇒ Precise Predictions



Precise predictions for:

- typology → e.g. no center embedding in phonology
- learnability → e.g. no Gold learning for regular languages
- cognition → e.g. finitely bounded working memory

Classifying Patterns

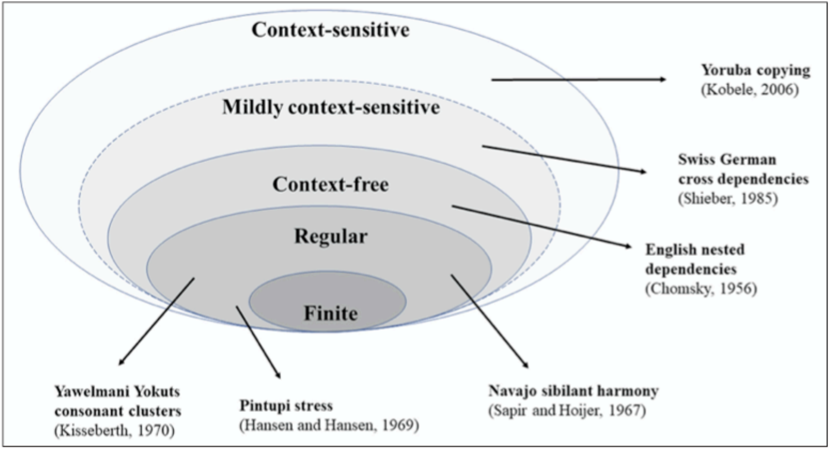
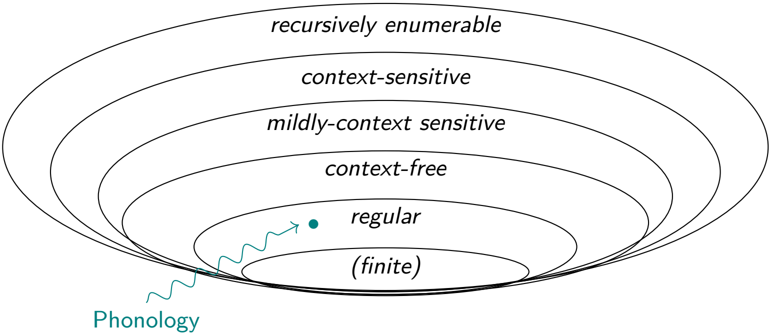


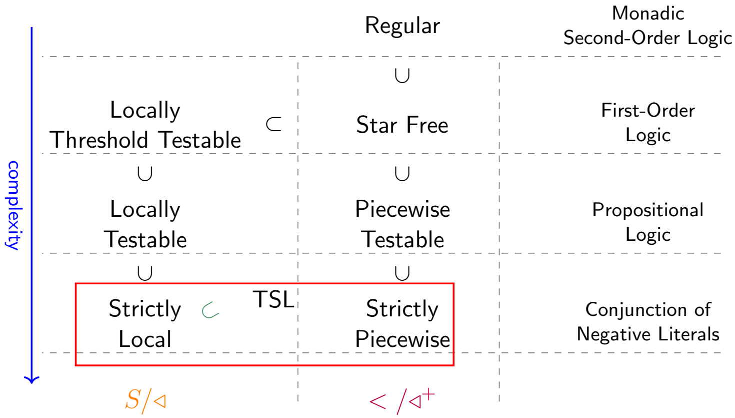
Figure 1: The Chomsky Hierarchy. Various features of natural language occupy different regions of the hierarchy. Figure reproduced from Figure 1 in Heinz (2010: 634) with permission.

Phonology as a Regular System



Kaplan and Kay (1994)

Beyond Monolithic Classes: Subregular Languages



- Multiple equivalent characterizations:
  $\Rightarrow$ algebraic, logic, automata...

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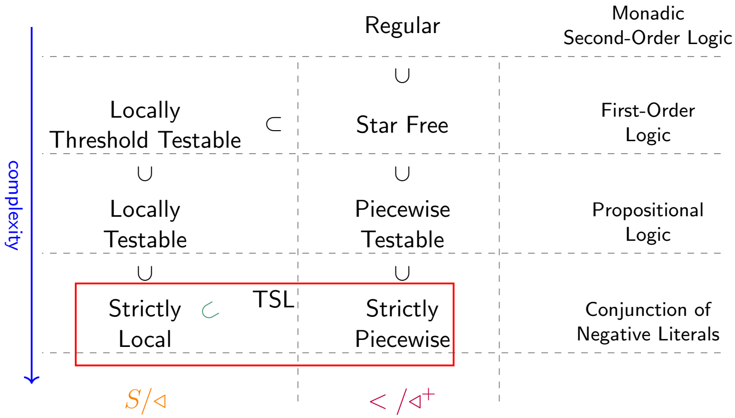

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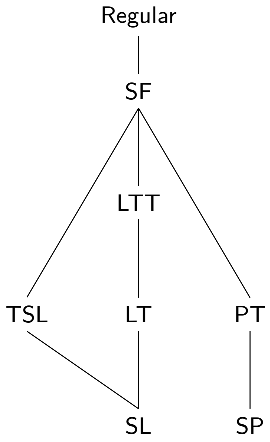
Phonology as a Subregular System

Subregular Phonotactics

- Majority of phonological patterns are **subregular** (Heinz 2011a,b; Chandlee 2014; Graf 2017:a.o.).

Most phonological and morphological rules correspond to p-subsequential relations.

(Mohri 1997)



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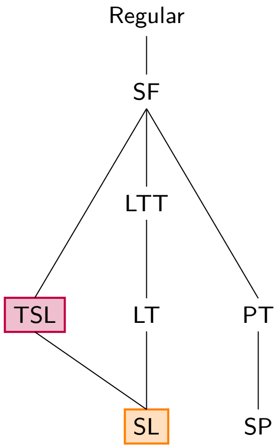
A caveat:
Mostly phonotactics today!



Local Dependencies in Phonology

1. **Word-final devoicing**
   Forbid voiced segments at the end of a word

(1) a. * rad
    b. rat

1. **Intervocalic voicing**
   Forbid voiceless segments in between two vowels

(2) a. * faser
    b. fazer

These patters can be described by **strictly local** (SL) constraints.

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Example: Word-final devoicing

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- **German**: ***z**$, ***v**$,***d**$ ($ = word edge).

$ r a **d** $ $ r a t $

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Unbounded Dependencies Are Not SL

- **Samala Sibilant Harmony**
  Sibilants must not disagree in anteriority.
  (Applegate 1972)

(3) a. * hasxintilawaʃ
b. * haʃxintilawas
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Example: Samala

* $ h a s x i n t i l a w a ʃ $

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Locality Over Tiers

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- **Problem**: SL limited to locality domains of size n;

Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- Projection of selected segments on a tier T (Goldsmith 1976)
- Strictly local constraints over T determine wellformedness
- Unbounded dependencies are local over **tiers**

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Unbounded Dependencies are TSL

- Let's revisit Samala Sibilant Harmony

  (4) a. * ha**s**xintilawaʃ

  b. * haʃxintilawa**s**

  c. haʃxintilawaʃ

- What do we need to project? [+strident]
- What do we need to ban? *[+ant][−ant],*[−ant][+ant]

  I.E. *sʃ, *sʒ, *zʃ, *zʒ, *ʃs, *ʒs, *ʃz, *ʒz

Example: TSL Samala

s ʃ

* $ h a [s] x i n t i l a w [ʃ] $

ʃ ʃ

ok $ h a [ʃ] x i n t i l a w [ʃ] $

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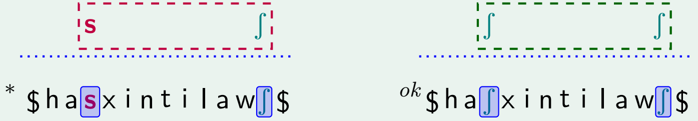
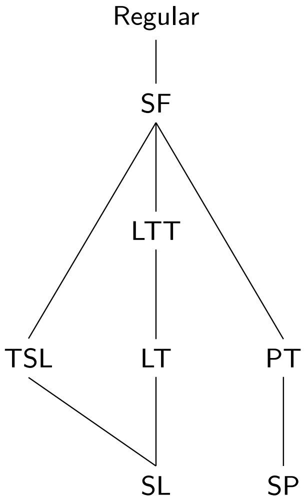


Interim Summary: SL and TSL for Phonology

- Linguistically natural (Goldsmith 1976)
- Captures wide range of phonotactic dependencies (McMullin 2016)
- Provably correct and efficient learning algorithms (Jardine and McMullin 2017)
- Rules out unattested patterns (cf. Lai 2015, Aksenova et al. 2016, Graf & De Santo 2019, a.o.)

Outline

SL and TSL: So What?



- **But** not every long-distance pattern is TSL!
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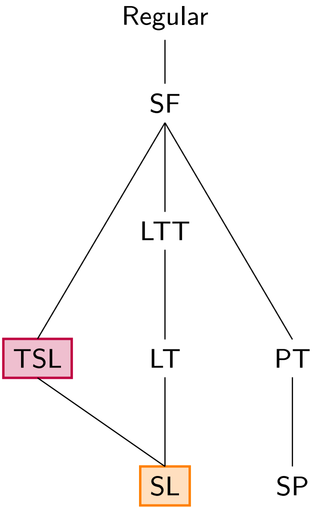

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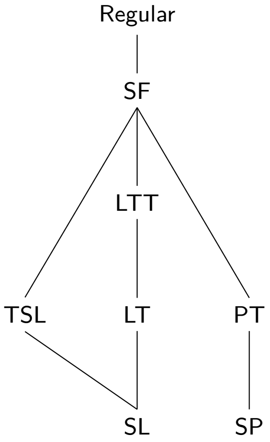
Concurrent Processes (De Santo and Graf, 2019)



Observation

- TSL is not closed under intersection

- We want to also account for multiple processes
  So we can cover the complete phonotactics of a language
- Multiple non-interacting processes in attested patterns

A TSL Outlier

Sibilant Harmony in Imdlawn Tashlhiyt (McMullin2016)

1) Underlying causative prefix /s(ː)-/

	Base	*Causative*	
a.	uga	sː-uga	"be evacuated"
b.	asːtwa	s-asːtwa	"settle, be levelled"

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2) Sibilant harmony

	Base	*Causative*	
a.	fiaʃr	ʃ- fiaʃr	"be full of straw, of discord"
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3) Sibilant voicing harmony blocked

	Base	*Causative*	
a.	ukz	sː-ukz	"recognize"
b.	qːuʒːi	ʃ- quʒːi	"be dislocated, broken"

Sibilant Harmony in Imdlawn Tashlhiyt

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$T = \{$ ʒ, s, z,ʃ$\}$
$S = \{$ *sʒ, *sz, *sʃ, *ʒs,*ʃs, *zs, *zʃ, *zʒ, *ʃz, *ʃʒ, *ʒʃ, *ʒz $\}$

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Voiceless obstruents block agreement in voicing.

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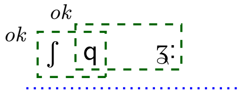

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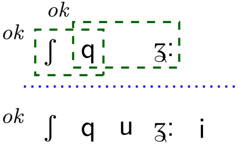

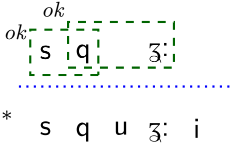

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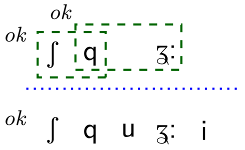

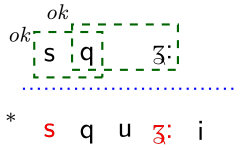

Multi-Tier Strictly Local (MTSL) Languages (1/2)

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Voiceless obstruents block agreement in voicing:

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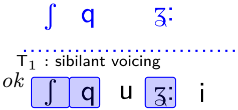

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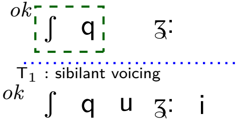

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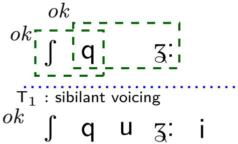

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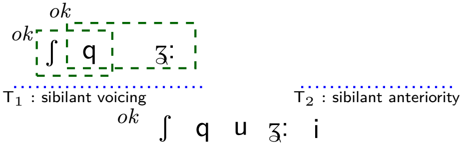
- T_2 = {ʒ, s, z,ʃ} S_2 = {*sʒ, *sʃ, *ʒs,*ʃs, *zs, *zʃ, *zʒ, *ʃz, *ʒz }



Multi-Tier Strictly Local (MTSL) Languages (1/2)

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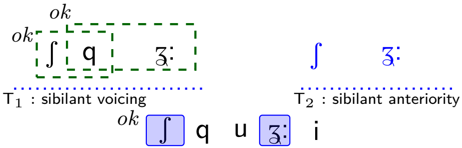

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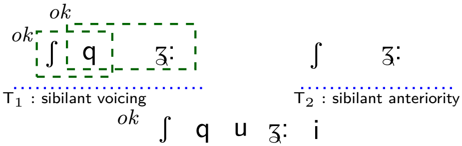

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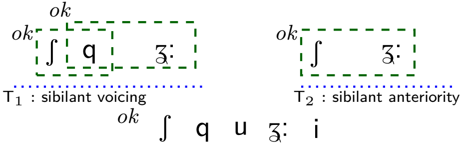

Multi-Tier Strictly Local (MTSL) Languages (2/2)

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* s q u ʒ: i

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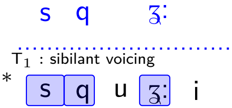

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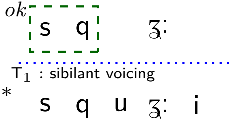

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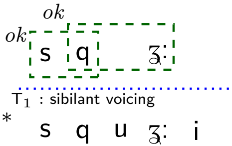

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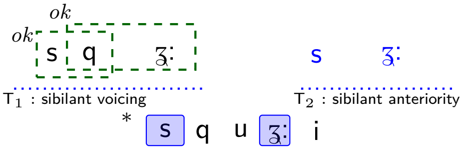

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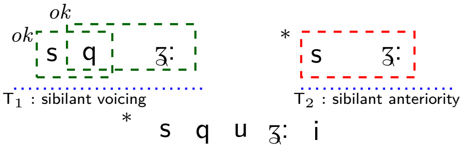
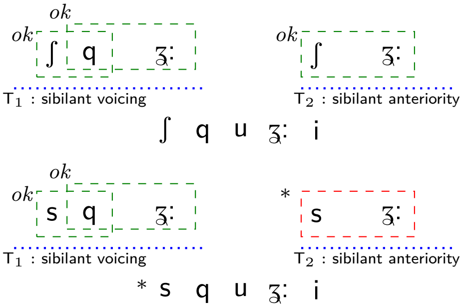


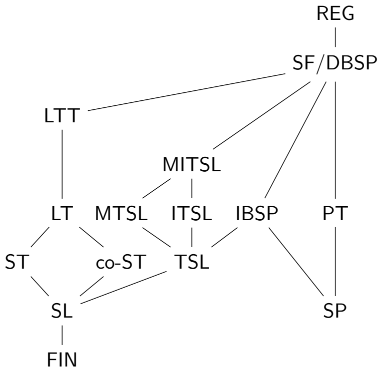
Accounting for Concurrent Processes

- MTSL: TSL closure under intersection
  (De Santo & Graf, 2019)



- Intersection closure accounts for multiple concurrent processes
- Can characterize the complete phonotactics of a language

A Plethora of Combination



- The goal is **not** identifying a single “correct” class
- Pinpoint fundamental properties of the patterns:
  SL: $\triangleleft$, TSL: $\triangleleft_T$, ...

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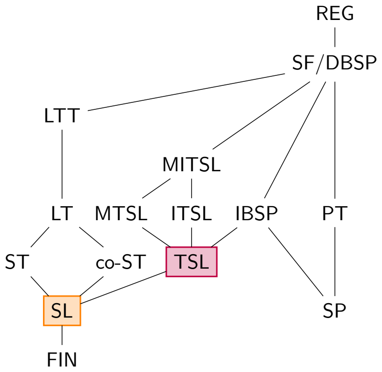

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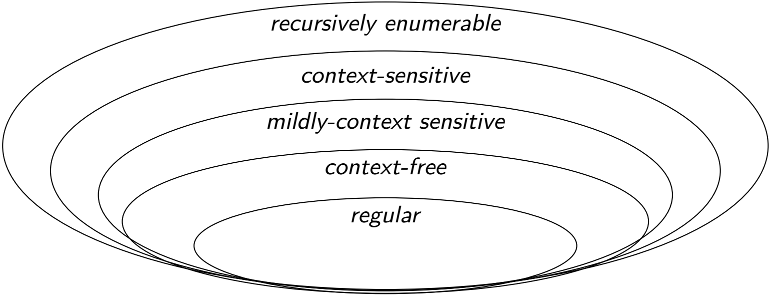
Cross-domain Parallels



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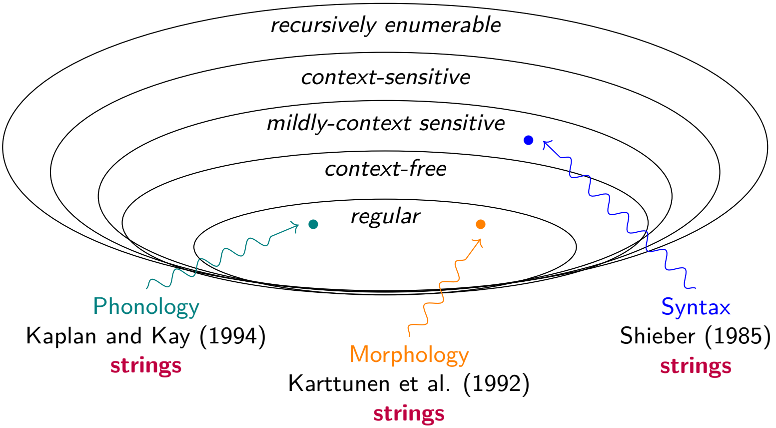

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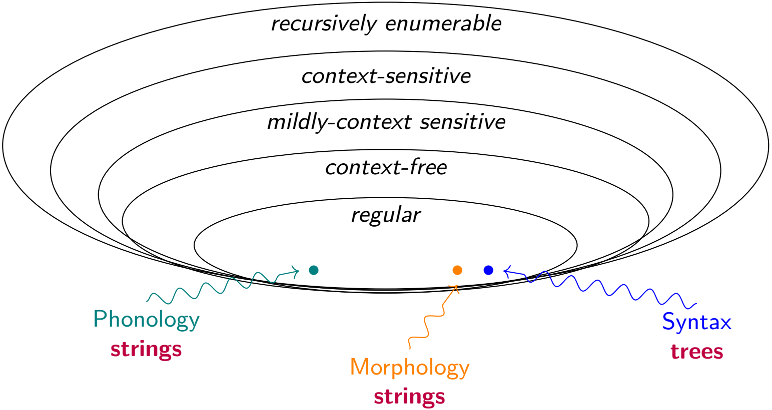
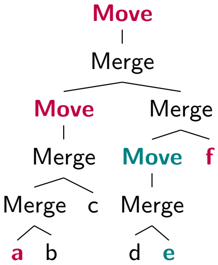


Subregular Syntax



- Some results for syntax
  - regular tree languages (Michaelis 2004; Kobele et al. 2007)
  - subregular operations (Graf 2018)
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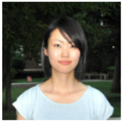

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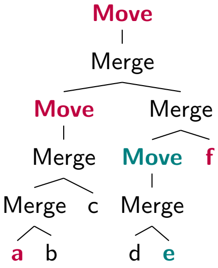

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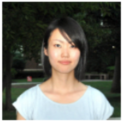

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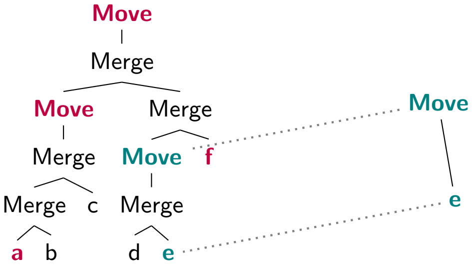

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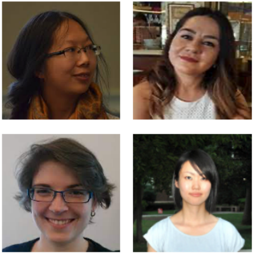

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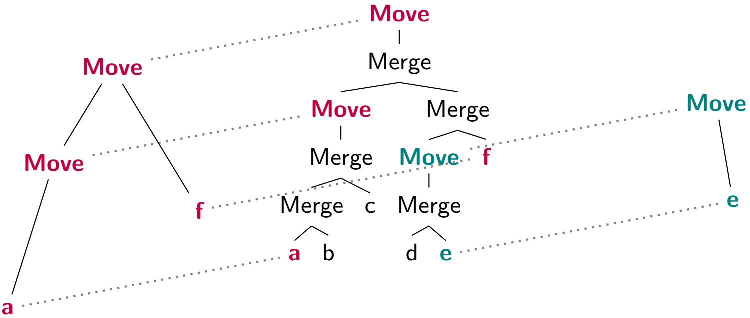

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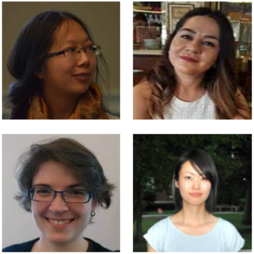

Interim Summary: Again, So What?

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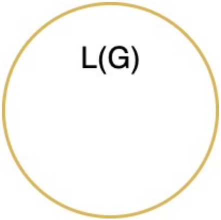
  Learnable from positive examples of strings/trees.

  Which information primitives are we sensitive to?

Outline

Artificial Grammar Learning (AGL)

- Can be used to test implicit learning abilities (Reber, 1976)



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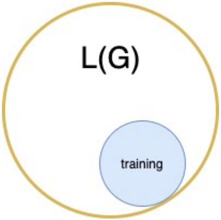

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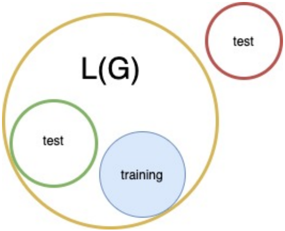

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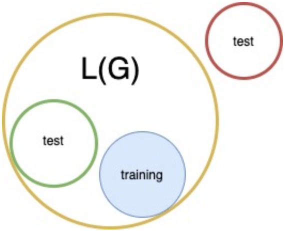

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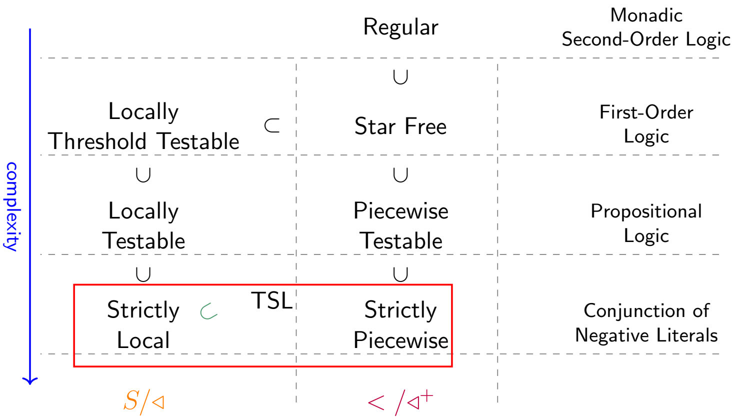
Testing Subregular Predictions



Example: Attested vs. Unattested Patterns

Attested: Unbounded Sibilant Harmony

- Every sibilant needs to harmonize

s ʃ

∗ $ h a s x i n t i l a w ʃ $

ʃ ʃ

ok $ h a ʃ x i n t i l a w ʃ $

Unattested: First-Last Harmony

- Harmony only holds between initial and final segments

s ʃ

ok $ h a s x i n t i l a w ʃ $

s ʃ

∗ $ s a t x i n t i l a w ʃ $

Lai (2015)

Learnable vs. Unlearnable Harmony Patterns

Regine Lai

Posted Online July 09, 2015
https://doi.org/10.1162/LING_a_00188

Linguistic Inquiry
Volume 46 | Issue 3 | Summer 2015
p.425-451

Keywords: phonotactics, learnability, computational phonology, formal theory, typology, dependencies

Lai (2015): Stimuli

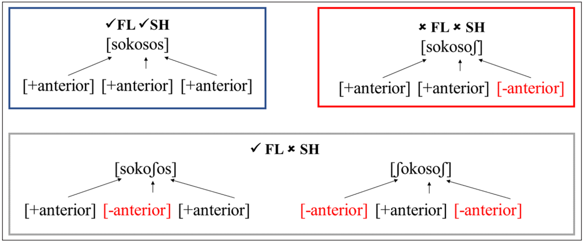


Figure 3: Comparison of SH and FL stimuli.

Lai (2015): Stimuli

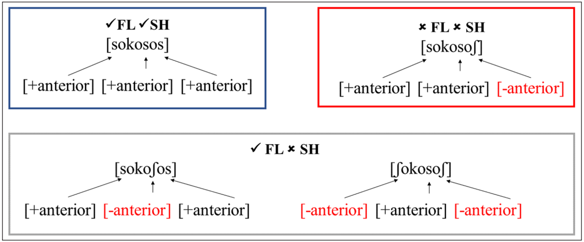
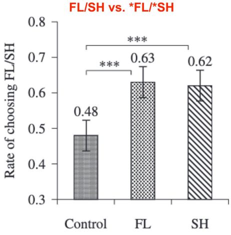


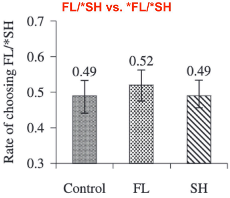
Figure 3: Comparison of SH and FL stimuli.

Table 6
Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

	Pairs		
Conditions	FL/*SH vs. *FL/*SH (e.g., [s . . . ʃ . . . s] vs. [s . . . s . . . ʃ]) Rate of FL/*SH	FL/SH vs. *FL/*SH (e.g., [s . . . s . . . s] vs. [s . . . s . . . ʃ]) Rate of FL/SH	FL/SH vs. FL/*SH (e.g., [s . . . s . . . s] vs. [s . . . ʃ . . . s]) Rate of FL/SH
SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

Lai (2015): Results





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- See Avcu and Hestvik (2020), Avcu et al. (2019) for replications

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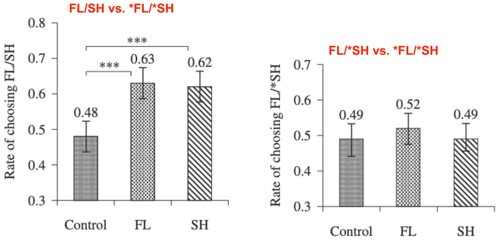

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A Plethora of Testable Predictions

Observation

- Attested patterns **A** and **B** are TSL.
- But combined pattern **A**+**B** is not TSL.

Prediction

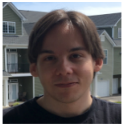
- **A**+**B** should be harder to learn than **A** and **B**

Example: Compounding Markers

Morphotactics as Tier-Based Strictly Local Dependencies

Alëna Aksënova Thomas Graf Sedigheh Moradi

- Russian has an infix *-o-* that may occur between parts of compounds.
- Turkish has a single suffix -sɪ that occurs at end of compounds.



(5)

vod	**-o-**	voz	**-o-**	voz
water	-COMP-	carry	-COMP-	carry

'carrier of water-carriers'

(6)

türk	bahçe	kapı	**-sɪ**	(***-sɪ**)
turkish	garden	gate	-COMP	(*-COMP)

'Turkish garden gate'

Example: Compounding Markers [cont.]

- Russian and Turkish are TSL.

Tier$_1$	COMP affix and stem edges #
Russian n-grams	**oo**, **$o**, **o$**
Turkish n-grams	**sisi**, **$si**, **si#**

- The combined pattern would yield **Ruskish**: stem^{n+1}-si^{n}
- This pattern is not regular and hence **not TSL either**.

Testable Predictions

- Can naive subjects learn Russian-like, Turkis-like, and Ruskish-like compounding?

Outline

1 Linguistics and Formal Language Theory

2 Refining the Hierarchy via Typological Insights

3 Artificial Grammar Learning

4 Summing Up & Future Directions

Of Black Swans and Flying Pigs

Of Black Swans and Flying Pigs

Of Black Swans and Flying Pigs

- Not a single data point, but classes of phenomena
- Value of restrictive theories: predictive and explanatory
- We learn from falsifying them too!

Complexity as a Magnifying Lens

- We can compare patterns and predictions across classes
- We can also compare patterns within a same class

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Formal Restrictions On Multiple Tiers

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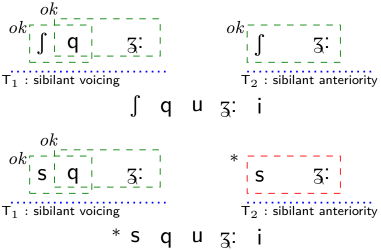
Testing Harmony Systems

Reminder:

- MTSL's multiple-tier idea...



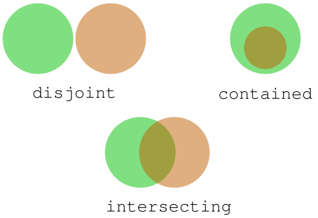


Figure 2: Theoretically possible tier alphabet relations

Testing Harmony Systems (cont.)

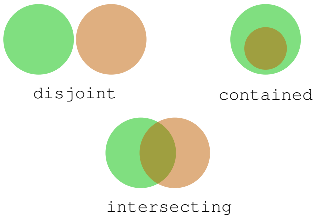
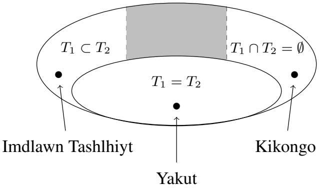


Figure 2: Theoretically possible tier alphabet relations



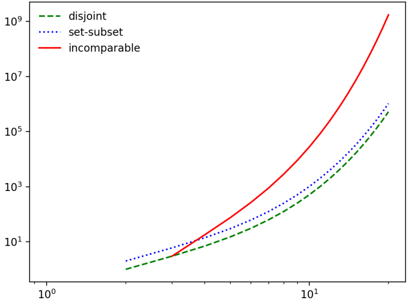


Figure 7: Growth of number of partitions of sets containing up to 20 elements (loglog scale)

Learnability Generalizations

Learning Interactions of Local and Non-Local Phonotactic Constraints from Positive Input

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- Efficiently learn MITSL_2^2 grammars from positive data

Unlearnable Patterns

- No overlapping tiers with the same $^*\rho_1\rho_2$ restriction
  e.g. $T_1 = \{a, b, c\},\ T_2 = \{a, b, d\},\ G_1 = G_2 = \{^*ab\}$

- This is *predicted* from the structure of the grammar (see also Lambert et al. 2021)

From Blackbox to Blackbox

Multi-Element Long Distance Dependencies: Using SP*k* Languages to Explore the Characteristics of Long-Distance Dependencies

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- Strictly-piecewise Languages
  - Basically: Skip-gram models
  - Capture long distance dependencies over strings
  - Modulate parameters of variation:
    e.g., length of the dependency, alphabet size, etc.

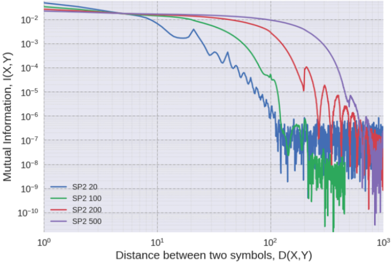
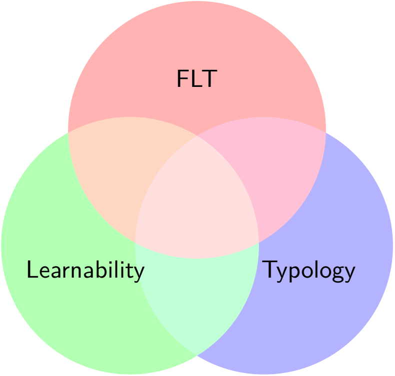


Figure 3: LDD characteristics of datasets of SP2 grammar exhibiting LDDs of length 20, 100, 200 and 500.

Theory Building

> *The problem that we cannot deduce [...] theories from data is a limitation, or* ***perhaps an attribute****, of all empirical science [...] Still, one may abduce hypotheses [...] Abduction is* ***reasoning from observations*** *[...] It consists of two steps: generating candidate* ***hypotheses*** *(abduction proper), and selecting the "best" explanatory one (inference to the* ***best explanation****).*
>
> *(van Roji & Baggio 2020, pg. 9)*

A Collaborative Enterprise!



Thank you!

Mathematical Linguistics and Cognitive Complexity
Aniello De Santo, Jonathan Rawski

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